Synoptic Meteorology II: Synoptic Development – The Pettersen-Sutcliffe Framework

10-12 March 2015

Readings: Sections 5.3.3 through 5.3.5 of *Midlatitude Synoptic Meteorology*.

Introduction

To this point in the semester, we have concentrated on understanding and applying four equations: the quasi-geostrophic vorticity equation, the quasi-geostrophic height tendency equation, the quasi-geostrophic omega equation, and the Q-vector form of the quasi-geostrophic omega equation. However, each of these equations is typically applied in the middle troposphere. While the movement and evolution of troughs and ridges above the surface is important, we are often interested in what is happening at the surface. In other words, how do surface cyclones and anticyclones evolve in response to synoptic-scale quasi-geostrophic forcing?

The quasi-geostrophic vorticity and omega equations form a system that can be used to interpret and describe the behavior of mid-latitude synoptic-scale weather systems. This is known as *Pettersen-Sutcliffe development theory*. In an earlier lecture, we hinted at the physical link between these two equations in the context of the destruction and subsequent restoration of geostrophic balance. In this lecture, we aim to describe precisely how these equations may be used to assess the synoptic-scale conditions that promote cyclone development (or cyclogenesis).

Obtaining the Pettersen-Sutcliffe Development Equation

Since the geostrophic relative vorticity ζ_g (by definition) and potential temperature θ (by application of the hydrostatic equation and Poisson's relation) can be written in terms of the geopotential height Φ, the quasi-geostrophic vorticity and omega equations form a system of two equations for two unknowns (Φ and ω), presuming that we know or can provide an estimate for the diabatic heating rate dQ/dt.

We start our derivation by recalling the quasi-geostrophic vorticity equation:

$$\frac{\partial \zeta_g}{\partial t} = -\vec{\mathbf{v}}_g \cdot \nabla \zeta_g - \beta v_g + f_0 \frac{\partial \omega}{\partial p} - K\zeta_g \tag{1}$$

If we are in a reference frame that is *moving* with the evolving synoptic-scale cyclone, the advection term – the first term on the right-hand side of (1) – does not contribute. Rather than reflecting the amplification of geostrophic relative vorticity, it merely reflects the movement of geostrophic relative vorticity.

The effect of the $-\beta v_g$ term, while non-zero, is quite small on the synoptic-scale. For typical values of β and v_g, we find this term to contribute to an approximate $1\text{x}10^{-5}$ s^{-1} increase in geostrophic relative vorticity over the course of 1 day. Observed synoptic-scale cyclones tend to deepen at a rate that is approximately one order of magnitude larger than this value. Therefore, we neglect this term.

Friction, as expressed by the fourth term on the right-hand side of (1), acts to weaken the intensity of synoptic-scale cyclones and anticyclones. Thus, for *surface cyclone development*, where the term on the left-hand side of (1) is positive, the stretching term – the third term on the right-hand side of (1) – must also be positive at and near the surface. Since f_0 is positive-definite in the Northern Hemisphere, we require that $\partial\omega/\partial p$ be positive at and near the surface.

When does this occur? First, we must assume that the vertical motion vanishes at the surface (i.e., $\omega_{sfc} = 0$). This implies that here is no vertical motion across the rigid surface of the ground. Since $\partial p < 0$, $\partial\omega$ must also be negative. With $\omega_{sfc} = 0$, $\omega_{aloft} < 0$. This means that there must be middle tropospheric synoptic-scale ascent for there to be surface cyclone development!

We use the quasi-geostrophic omega equation to evaluate the conditions under which there is middle tropospheric synoptic-scale ascent. This requires manipulating the quasi-geostrophic omega equation, given by Equation (2) below, to determine the value of $\partial\omega/\partial p$ at the surface ($p = p_{sfc}$).

$$\sigma\nabla^2\omega + f_0^2\frac{\partial^2\omega}{\partial p^2} = -f_0\frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g\cdot\nabla\left(\zeta_g + f\right)\right) - h\nabla^2\left(-\vec{\mathbf{v}}_g\cdot\nabla\theta\right) + f_0\frac{\partial}{\partial p}\left(K\zeta_g\right) - \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \qquad (2)$$

In the quasi-geostrophic omega equation, we have a term that looks similar to $\partial\omega/\partial p$, as given by $\partial^2\omega/\partial p^2$. We wish to rewrite (2) in terms of this term. In doing so, we will neglect the frictional and geostrophic horizontal advection of planetary vorticity terms. Likewise, we wish to express the geostrophic potential temperature advection term in terms of the geopotential height Φ via use of the form of the hydrostatic relationship given by:

$$\frac{\partial\Phi}{\partial p} = -h\theta \qquad (3)$$

Doing so, we obtain the following form of the quasi-geostrophic omega equation:

$$f_0^2\frac{\partial^2\omega}{\partial p^2} = -f_0\frac{\partial}{\partial p}\left(-\vec{\mathbf{v}}_g\cdot\nabla\zeta_g\right) - \nabla^2\left(-\vec{\mathbf{v}}_g\cdot\nabla\left(-\frac{\partial\Phi}{\partial p}\right)\right) - \sigma\nabla^2\omega - \frac{R}{pc_p}\nabla^2\left(\frac{dQ}{dt}\right) \qquad (4)$$

To get an expression for $\partial\omega/\partial p$, we wish to integrate (4) with respect to p. We do so from the surface ($p = p_{sfc}$) to the level of non-divergence (LND; $p = p_{LND}$). This can typically be found in the middle troposphere. The LND arises from Dines' compensation principle, which states that the vertically-integrated divergence in a given vertical column must be balanced by the vertically-integrated convergence in that column. Stated differently, this requires that the sign of the divergence change at least once within the column, resulting in one or more locations where the divergence is equal to zero (i.e., the LND).

Performing this integration, we obtain:

$$\begin{aligned} f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{LND}} - f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} &= -f_0 \left(-\vec{\mathbf{v}}_g \cdot \nabla \zeta_g\right)\Big|_{p_{LND}} + f_0 \left(-\vec{\mathbf{v}}_g \cdot \nabla \zeta_g\right)\Big|_{p_{sfc}} \\ &\quad - \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left(-\vec{\mathbf{v}}_g \cdot \nabla \left(-\frac{\partial \Phi}{\partial p}\right)\right) dp - \int_{p_{sfc}}^{p_{LND}} \left[\sigma \nabla^2 \omega + \frac{R}{p c_p} \nabla^2 \left(\frac{dQ}{dt}\right)\right] dp \end{aligned} \tag{5}$$

The first term on the left-hand side of (5) drops out by definition of the LND. Because the divergence is zero at the LND, from continuity, so too is $\partial\omega/\partial p$. Similarly, we neglect the second term on the right-hand side of (5) by presumption that the geostrophic vorticity advection at the surface is small. With this in mind, (5) becomes:

$$-f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} = -f_0 \left(-\vec{\mathbf{v}}_g \cdot \nabla \zeta_g\right)\Big|_{p_{LND}} - \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left(-\vec{\mathbf{v}}_g \cdot \nabla \left(-\frac{\partial \Phi}{\partial p}\right)\right) dp - \int_{p_{sfc}}^{p_{LND}} \left[\sigma \nabla^2 \omega + \frac{R}{p c_p} \nabla^2 \left(\frac{dQ}{dt}\right)\right] dp \tag{6}$$

Equation (6) is what we call the *Pettersen-Sutcliffe Development Equation* and relates the time rate of change of the surface geostrophic vorticity – as manifest through its tie to $\partial\omega/\partial p$ in the quasi-geostrophic vorticity equation – to a set of three forcing terms. We now wish to consider the contributions from each of these terms to surface cyclone development in isolation.

Interpretation of the Pettersen-Sutcliffe Development Equation

Let us first consider the case where the only forcing term in (6) is the geostrophic vorticity advection term, i.e.,

$$f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} \approx f_0 \left(-\vec{\mathbf{v}}_g \cdot \nabla \zeta_g\right)\Big|_{p_{LND}} \tag{7}$$

For cyclone development to occur, $\left(-\vec{\mathbf{v}}_g \cdot \nabla \zeta_g\right)\Big|_{p_{LND}}$ must be positive. By the sign convention associated with advection, this means that *there must be cyclonic geostrophic vorticity advection occurring at the LND in order for cyclone development to occur*!

Let us now consider the case where the only forcing term in (6) is the static stability and diabatic heating term, i.e.,

$$f_0^2 \frac{\partial \omega}{\partial p}\bigg|_{p_{sfc}} \approx \int_{p_{sfc}}^{p_{LND}} \left[\sigma \nabla^2 \omega + \frac{R}{pc_p} \nabla^2 \left(\frac{dQ}{dt}\right)\right] dp = \int_{p_{sfc}}^{p_{LND}} \sigma \nabla^2 \omega dp + \int_{p_{sfc}}^{p_{LND}} \frac{R}{pc_p} \nabla^2 \left(\frac{dQ}{dt}\right) dp \tag{8}$$

Let us first consider the static stability term, that involving σ. Recall that σ is a measure of the dry static stability, where $\sigma < 0$ implies static instability (or potential temperature decreasing with height) and $\sigma > 0$ implies static stability (or potential temperature increasing with height). We let $\sigma \nabla^2 \omega$ be approximated by a layer-mean value $\overline{\sigma \nabla^2 \omega}$ such that it is no longer a function of p. Thus,

$$\int_{p_{sfc}}^{p_{LND}} \sigma \nabla^2 \omega dp \approx \overline{\sigma \nabla^2 \omega} \int_{p_{sfc}}^{p_{LND}} dp = \overline{\sigma \nabla^2 \omega} \left(p_{LND} - p_{sfc}\right) \tag{9}$$

Since $p_{LND} - p_{sfc}$ is negative, for cyclone development to occur, $\overline{\sigma \nabla^2 \omega}$ must also be negative. Recall that there must be layer-mean ascent ($\overline{\omega} < 0$) for cyclone development to occur. In this case, since $\nabla^2 \omega \propto -\omega$, the Laplacian term is positive. Thus, for $\overline{\sigma \nabla^2 \omega}$ to be negative, σ must be negative, signifying static instability. However, dry static instability is not often present, such that σ is most often positive. As a result, we find that *ascent results in a decrease in cyclonic geostrophic relative vorticity at the surface over time, a* ***cyclolytic*** *situation*!

Next, let us consider the diabatic heating term. We presume that the diabatic heating can also be represented by a layer mean value such that it is no longer a function of p. Thus, we write:

$$f_0^2 \frac{\partial \omega}{\partial p}\bigg|_{p_{sfc}} \approx \frac{R}{c_p} \nabla^2 \left(\overline{\frac{dQ}{dt}}\right) \int_{p_{sfc}}^{p_{LND}} d\ln(p) = \frac{R}{c_p} \nabla^2 \left(\overline{\frac{dQ}{dt}}\right) \ln\left(\frac{p_{LND}}{p_{sfc}}\right) \tag{10}$$

Since $p_{LND} < p_{sfc}$, the natural logarithm term is negative. Thus, for surface cyclone development, we require that $\nabla^2 \left(\overline{\frac{dQ}{dt}}\right) < 0$. Since $\nabla^2 \left(\overline{\frac{dQ}{dt}}\right) \propto -\overline{\frac{dQ}{dt}}$, this requires that $\overline{\frac{dQ}{dt}} > 0$. Thus, we find that *diabatic warming contributes positively to surface cyclone development*!

It is worth noting that diabatic warming generally acts to decrease the static stability σ. As a result, diabatic warming has both direct and indirect positive contributions to surface cyclone development. This allows us to state that the presence of moisture, associated with diabatic heating such as manifest through latent heat release, can substantially aid surface cyclone development!

Finally, let us consider the case where the only forcing term in (6) is the geostrophic advection of the partial derivative of geopotential height with respect to pressure, i.e.,

$$f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} \approx \nabla^2 \int_{p_{sfc}}^{p_{LND}} \left(-\vec{\mathbf{v}}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p} \right) \right) dp \tag{11}$$

If we make the assumption that the *direction* of the potential temperature gradient $\nabla\theta$, recalling that $\nabla(h\theta) = \nabla\left(-\frac{\partial \Phi}{\partial p} \right)$, is *constant* with respect to height, such that the isotherms are oriented in the same direction on each isobaric level between the surface and LND, we find that:

$$-\vec{\mathbf{v}}_g \cdot \nabla\left(-\frac{\partial \Phi}{\partial p} \right) \approx -\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}} \cdot \nabla\left(-\frac{\partial \Phi}{\partial p} \right) \tag{12}$$

Since $\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}}$ is constant with respect to pressure, this allows us to write:

$$f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} \approx \nabla^2 \left(-\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}} \cdot \nabla\left(-\int_{p_{sfc}}^{p_{LND}} \left(\frac{\partial \Phi}{\partial p} dp \right) \right) \right) \tag{13}$$

If we take the integral represented in (13), we obtain:

$$f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} \approx -\nabla^2 \left(-\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}} \cdot \nabla\left(\Phi_{p_{LND}} - \Phi_{p_{sfc}} \right) \right) \tag{14}$$

In (14), $\Phi_{p_{LND}} - \Phi_{p_{sfc}}$ is the geopotential thickness of the layer between $p = p_{LND}$ and $p = p_{sfc}$. From the known relationship between the geopotential thickness of a vertical layer and the mean potential temperature of that layer, we can alternatively express (14) as:

$$f_0^2 \left.\frac{\partial \omega}{\partial p}\right|_{p_{sfc}} \approx -\nabla^2 \left(-\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}} \cdot \nabla\overline{\theta} \right) \tag{15}$$

In (15), the overbar on θ indicates a vertical average between p_{LND} and p_{sfc}.

For surface cyclone development, given the sign convention on the Laplacian operator in (15), we desire to find where $-\left.\vec{\mathbf{v}}_g\right|_{p_{sfc}} \cdot \nabla\overline{\theta} > 0$. This term represents the advection of layer-mean potential temperature by the surface geostrophic wind. Based upon the sign convention on advection, we find that *this – and, thus, surface cyclone development – occurs when there is warm layer-mean potential temperature advection*!

Implicitly, as this term is non-zero only when there is a horizontal gradient of potential temperature, which is a measure of the *baroclinicity* of the atmosphere, we expect that cyclone development occurs in the presence of non-zero baroclinicity (i.e., in the presence of a horizontal potential temperature gradient).

One caveat regarding the interpretation of this term, however. Because of the assumption that the direction of the horizontal potential temperature gradient is constant with height, we approximate the advection term with the advection by the surface geostrophic wind. However, the surface geostrophic wind vanishes at the center of a cyclone (or anticyclone, for that matter). Thus, there is no net advection over the center of a surface cyclone in as much as this assumption – as well as all of those inherent to the quasi-geostrophic system – holds.

Thus, we state that this term is not responsible for surface cyclone *intensity changes* but, rather, is responsible primarily for surface cyclone *motion*. As a surface cyclone will generally track towards regions favoring development, surface cyclones move toward areas of layer-mean warm potential temperature advection and away from areas of layer-mean cold potential temperature advection. In the Northern Hemisphere, for cyclonic rotation, layer-mean warm potential temperature advection is typically found to its north and east whereas layer-mean cold potential temperature advection is typically found to its south and west. This is depicted in Figure 1 below.

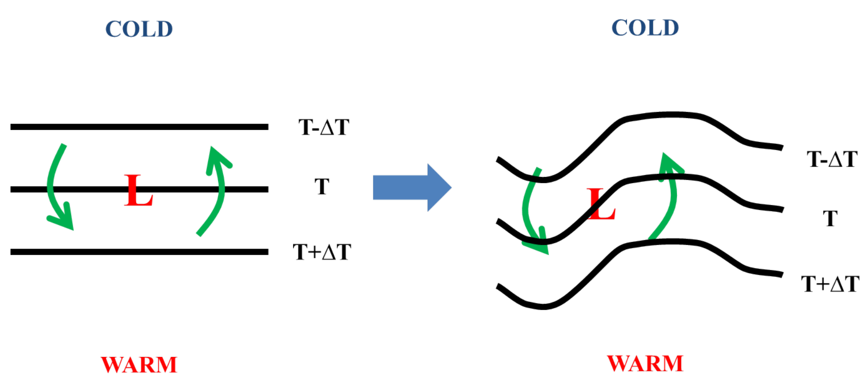

Figure 1. Idealized schematic of the rotation of layer-mean isotherms (black lines) by the cyclonic geostrophic flow associated with a surface cyclone (green lines). The cyclonic geostrophic flow results in warm (cold) advection to the north/east (south/west).

<u>The Concept of "Self Development"</u>

If we consider the interpretations of each forcing term together, we can state:

- Surface cyclone development occurs when and where meaningful cyclonic geostrophic vorticity advection in the middle to upper troposphere becomes superimposed upon a quasi-stationary surface front (or baroclinic zone).

- When there is diabatic warming, such as is often found with latent heat release associated with condensation and precipitation, such development can be more rapid and/or intense.

- Surface cyclones move toward regions of development, as largely manifest through thermal advection patterns associated with the cyclone itself.

These findings provide the basic framework for what is known as the "self-development" paradigm for cyclone development. The remainder of this lecture will illustrate this in the context of the full life cycle of a synoptic-scale surface cyclone. Note that in the following, we assume that there is westerly vertical wind shear in approximate thermal wind balance with the horizontal temperature gradient and that we are in a frame of reference moving with the system.

Step 1: Surface Cyclogenesis

In Step 1, an upper tropospheric trough approaches a lower tropospheric baroclinic zone, such as that associated with a remnant frontal boundary. Ahead of the trough, there is cyclonic geostrophic vorticity advection. Behind it, there is anticyclonic geostrophic vorticity advection. If we presume that the trough is at or near the LND, then from our interpretation of (7), there should be ascent and surface cyclone development ahead of the trough and descent and surface anticyclone development behind the trough. This leads to the initial development of a surface cyclone (as well as anticyclone).

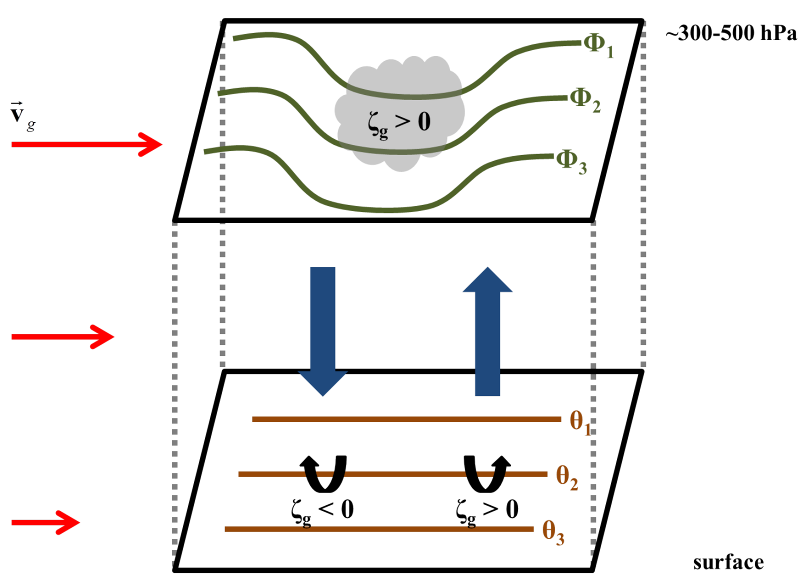

Figure 2. Idealized schematic of the upper and lower tropospheric pattern and accompanying forcings associated with the genesis stage of a synoptic-scale surface cyclone. The upper tropospheric pattern is at or near the level of the LND (roughly 500 hPa). In this and the schematics presented in Figures 3, 4, and 6, contours aloft represent lines of constant geopotential; contours at the surface represent lines of constant potential temperature. Filled arrows denote vertical motion forcing due to geostrophic vorticity advection at the LND; open arrows denote vertical motion forcing due to thermal advection at the surface. WAA and CAA denote warm and cold potential temperature advection, respectively.

Step 2: Surface Development

In Step 2, at some unspecified later time, the surface flow associated with the nascent surface cyclone and anticyclone acts to modify the orientation of the lower tropospheric isotherms from a near-zonal orientation to one with wave-like structure. This is indicative of cold potential temperature advection between the surface anticyclone and surface cyclone and warm potential temperature advection to the north and east of the surface cyclone. From our interpretation of (15), this causes *mid-tropospheric ascent and surface pressure falls to the north and east of the surface cyclone* and *mid-tropospheric descent and surface pressure rises to the south and west of the surface cyclone*. This results in the northeastward movement of the surface cyclone and southeastward movement of the surface anticyclone.

The aforementioned cold layer-mean potential temperature advection is found beneath the upper tropospheric trough. Likewise, the aforementioned warm potential temperature advection is found beneath the upper tropospheric ridge. From our interpretation of the quasi-geostrophic height tendency equation, presuming that both potential temperature advections are maximized near the surface and decay with height, height falls occur in the base of the trough and height rises occur in the apex of the ridge, strengthening the upper level pattern by amplifying the trough and ridge and, typically, shortening the wavelength. This acts to amplify the geostrophic relative vorticity and its advection associated with the trough-ridge pattern.

The continued – and enhanced – geostrophic relative vorticity advection at the LND works to further intensify the surface features via the same mechanisms noted in Step 1.

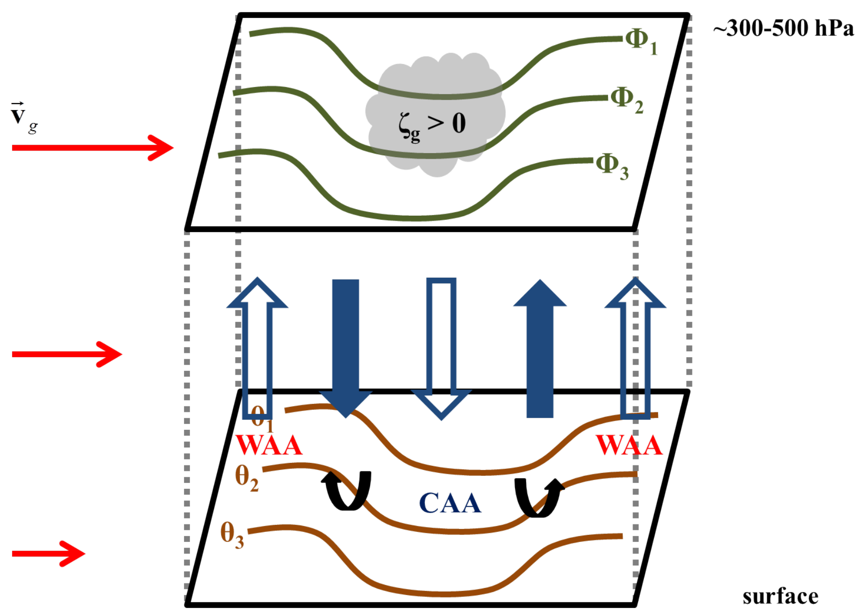

Figure 3. Idealized schematic of the upper and lower tropospheric pattern and accompanying forcings associated with the early development stage of a synoptic-scale surface cyclone.

The key take-home points to this point are as follows:

- The pattern of geostrophic vorticity advection associated with the upper tropospheric features acts to intensify surface features.
- The pattern of thermal advection (and the accompanying vertical motions) associated with the surface features acts to intensify the upper tropospheric features.
- A feedback loop thus exists as both sets of features move eastward.
  - The eastward motion of the surface features is driven by the thermal advection forcing described above.
  - The eastward motion of the upper tropospheric features is driven by advection, as elucidated via our interpretation of the geostrophic vorticity advection term of the quasi-geostrophic height tendency equation.

The fact that there is an upshear tilt, or one to the west with increasing height against the vertical wind shear, promotes this feedback loop. If this tilt remains constant, it is known as *phase locking*. You will likely hear more about phase locking in your study of waves in Dynamics II. If the tilt changes, the evolution of the pattern will change as the forcings evolve in both location and intensity. We will see this in action as we progress forward in time.

Step 3: Development to Maturity

The intensification of the upper tropospheric trough has intensified the geostrophic relative vorticity advection aloft, thereby enhancing its associated patterns of ascent and descent and promoting the further intensification of the surface features. The intensified surface features continue to rotate the isotherms via advection, as discussed previously.

With warm potential temperature advection maximized further northward with time, the surface cyclone takes on an increasingly large poleward (northward) component of motion. Likewise, with cold potential temperature advection maximized further southward with time, the surface anticyclone takes on an increasingly large equatorward (southward) component of motion. While these surface features may appear to move in the same direction as the upper tropospheric winds, this is mere coincidence – their motion is instead being driven by lower tropospheric thermal advection.

Aloft, the reorientation of the maxima of lower tropospheric thermal advection results in a concordant reorientation of where the forcing for middle tropospheric height falls and rises is located: falls in the southern and eastern portion of the upper trough, rises in the northern and western portion of the upper ridge. This leads to both the trough and ridge becoming *negatively tilted* (i.e., from northwest to southeast in the horizontal plane). This is depicted in Figure 5.

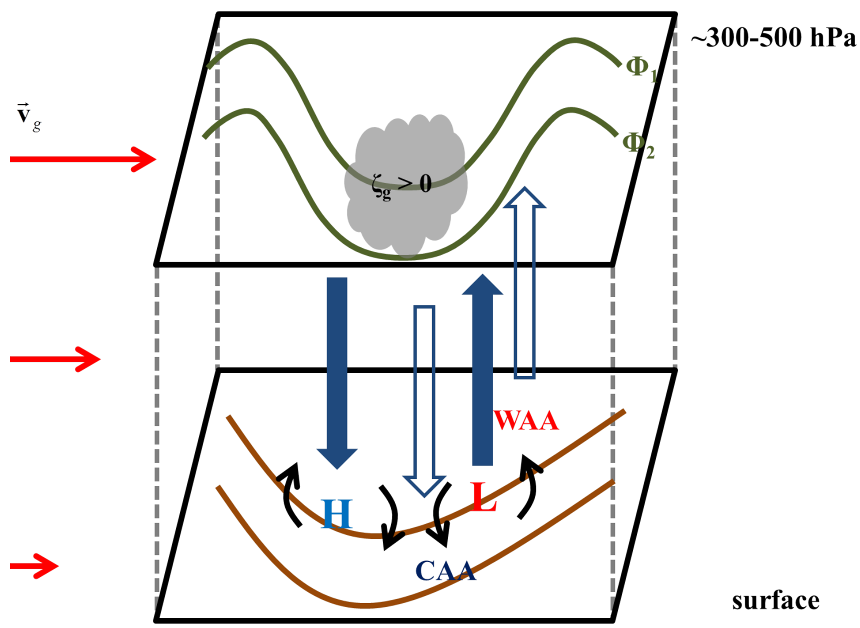

Figure 4. Idealized schematic of the upper and lower tropospheric pattern and accompanying forcings associated with the mature development stage of a synoptic-scale surface cyclone.

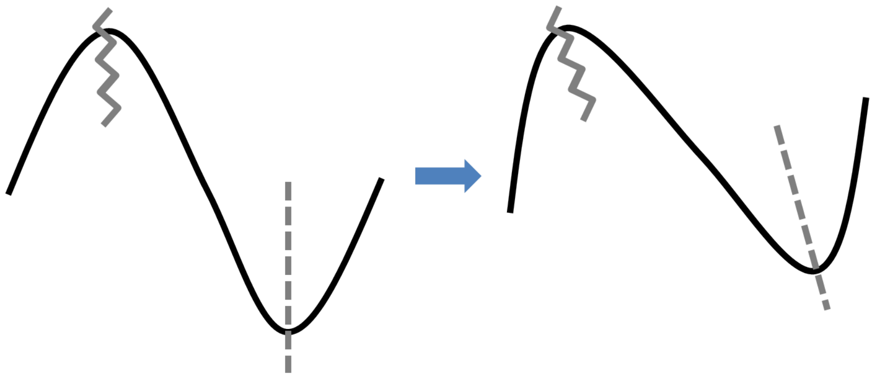

Figure 5. Idealized depiction of the transition from a “neutrally-tilted” ridge/trough pattern early in a system’s lifecycle (at left) to one featuring a “negatively-tilted” ridge/trough pattern as the system approaches maturity (at right).

Step 4: From Maturity to Occlusion

As the system matures toward occlusion (and its peak intensity), the lower tropospheric isotherm pattern becomes further distorted into an “S” shape, leading to warm thermal advection primarily north of the surface cyclone and cold thermal advection primarily south of the surface cyclone. This brings about even more of a northward motion of the surface cyclone.

The amplification of and increasing negative tilt to the upper tropospheric ridge/trough pattern weakens the zonal component of the geostrophic relative vorticity advection aloft, slowing the eastward movement of the upper tropospheric pattern. However, given the primarily poleward motion of the surface features described above, this leads to the tilt between the surface and upper level features growing smaller with time.

As a result, there is an increasingly large degree of overlap between the forcings for ascent and descent promoted by the lower and upper tropospheric features. This helps the surface cyclone to reach its maximum intensity. However, as the magnitude of the advections gradually weakens both aloft and at the surface due to the amplification and tilt of the upper level pattern and reorientation of the lower tropospheric isotherms, respectively, the magnitude of these forcings weakens.

Eventually, the lower tropospheric thermal field becomes substantially distorted as it wraps around the cyclone such that there is little potential temperature advection anywhere. Thus, the forcing upon the upper tropospheric trough’s intensity and the surface cyclone’s movement goes away. Friction acts to spin down the surface cyclone, while the upper tropospheric trough may cut off from the synoptic-scale westerly flow as a result of its amplification. Without any forcing remaining to intensify it, it too will gradually decay unless or until some other forcing is imposed.

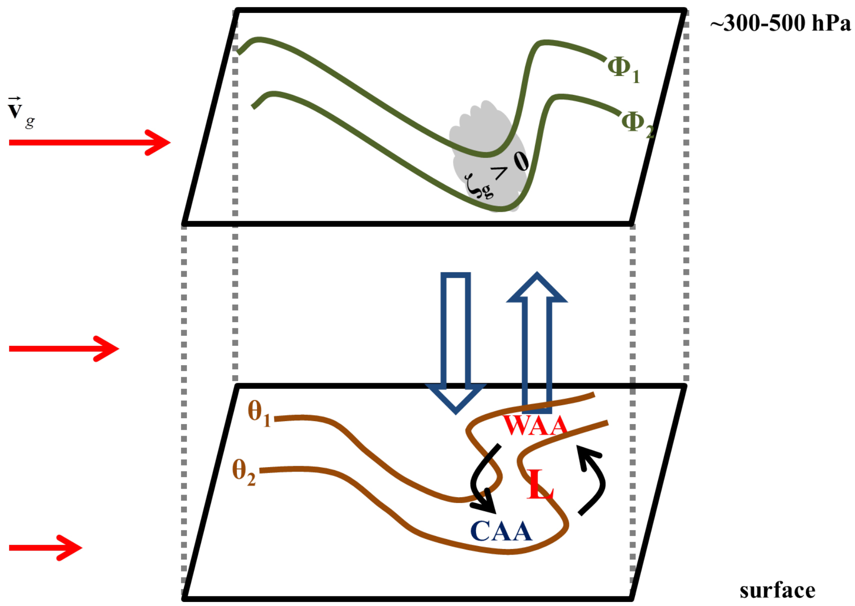

Figure 6. Idealized schematic of the upper and lower tropospheric pattern and accompanying forcings associated with the mature stage of a synoptic-scale surface cyclone.

Thus, the closed system provided by the *quasi-geostrophic vorticity and omega equations*, as manifest through the *Pettersen-Sutcliffe development equation*, is sufficient to describe surface cyclone formation. When coupled with the *quasi-geostrophic height tendency equation*, the entirety of a synoptic-scale cyclone and upper tropospheric trough's coupled lifecycle can be described. In our next set of notes, we will examine this lifecycle in the context of an intense surface cyclogenesis event over the North Atlantic Ocean from January 2013.