THE PHYSICS OF SUPERFLUID HELIUM

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Abstract
The paper contains a brief account of the physics of superfluid ^{4}He, with emphasis on the underlying physical principles; it uses the minimum of mathematics, and there is some emphasis on aspects that relate to practical applications.

1. INTRODUCTION AND HISTORY

Helium was first liquefied by Kammerlingh Onnes in Leiden in 1908. During the late 1920s and early 1930s it was noticed that the liquid had some strange properties, but it was not until 1938 that it was discovered independently by Allen and Misener and by Kapitza that it exhibited frictionless flow and was what we now call a superfluid. Shortly afterwards Fritz London suggested that superfluidity could have some connection with Bose-Einstein condensation, which was known as a theoretical possibility in an ideal Bose gas. London also realized that there might be a strong connection with superconductivity, which had been discovered many years before and which could be seen as superfluidity in the electron gas in a metal. With impressive intuition he also suggested that both superfluidity and superconductivity were "quantum mechanisms on a macroscopic scale", although the significance of this idea did not become really clear until the late 1950s or early 1960s.

Shortly after London produced these seminal ideas he and Tisza suggested that the superfluid phase of the liquid could be described by a two-fluid model, the condensed and non-condensed atoms being identified respectively with the superfluid and normal components. In 1941 Landau wrote a remarkable paper in which he suggested that superfluidity can be understood in terms of the special nature of the thermally excited states of the liquid: the well-known phonons and rotons. This idea led Landau also to the idea of a two-fluid model, but with a microscopic interpretation that was different from that of London and Tisza. Indeed, Landau expressed the view that superfluidity has no obvious connection with Bose condensation, although, as we shall see, this view was certainly wrong. Nevertheless, the basic ideas in Landau's paper were correct, and his interpretation of the two-fluid model showed brilliant intuition.

After the second world war the two-fluid model was placed on a firm experimental basis, especially with the experiment of Andronikashvili and the discovery of second sound. At the same time the properties of the normal fluid (the gas of phonons and rotons) were explored in great theoretical detail by Khalatnikov, with parallel confirmatory experiments.

A theoretical proof that Bose condensation does occur in a liquid such as superfluid helium was provided by Onsager and Penrose. Feynman wrote a number of important papers in the 1950s, exploring how the properties of liquid helium were strongly related to the fact that the atoms obey Bose statistics. The quantization of superfluid circulation and the existence of free quantized vortices were proposed theoretically and independently by Onsager and Feynman, and the first experimental confirmation came from the work of Hall and Vinen with the discovery of mutual friction in rotating helium and with the direct observation in a macroscopic experiment of the quantization of circulation. This work led to an appreciation for the first time of the full significance of London's "quantum mechanism on a macorscopic scale", and of the underlying importance of Bose condensation in superfluidity.

In 1957 Bardeen, Cooper and Schrieffer wrote their famous paper on the theory of superconductivity. In due course this led to a better appreciation of the connection between superfluidity and superconductivity, and the discovery of the quantization of flux and of free flux lines in type II superconductors demonstrated clearly the analogies between the two systems. As far as we know all superfluids and superconductors have one basic feature in common: their properties derive from the existence within them of some type of Bose condensation, involving atoms or pairs of atoms or pairs of electrons.

Liquid ^{3}He exhibits no superfluid behaviour at the relatively high temperatures involved in superfluid ^{4}He, thus confirming the importance of particle statistics in this behaviour. The discovery of superfluidity in liquid ^{3}He by Osheroff, Richardson and Lee in 1973 at a temperature of about 2mK completed the story, showing that BCS pairing can occur in an uncharged Fermi liquid; the pairs are now pairs of atoms, but the pairing is unconventional in that it involves relative p-states rather than the s-states of the conventional BCS theory. Unconventional pairing is now known to occur in exotic superconductors, such as the heavy-fermion metals and the high-temperature materials.

In these brief notes we shall focus our attention on superfluidity in liquid ^{4}He, emphasizing the underlying physical principles, including those associated with macroscopic quantum phenomena, and we shall place some emphasis on aspects that relate to practical applications.

The following references contain useful introductory reading [1], [2], [3].

2. THE PHASE DIAGRAM OF ^{4}He

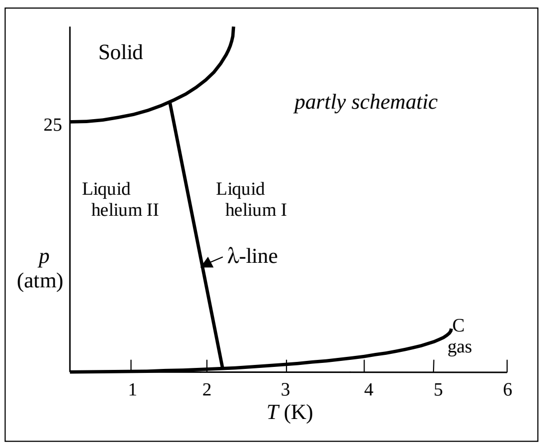

Fig. 1: The phase diagram of ^{4}He.

We see that the phase diagram in Fig. 1 exhibits two anomalous features. The liquid phase exists over a range of pressure up to about 25 atm even at the absolute zero of temperature; and there are two liquid phases, helium I, which is conventional in its properties, and helium II, which is superfluid.

The existence of a liquid over a range of pressures at $T = 0$ must be a quantum effect. It arises from quantum mechanical zero point energy: the fact that a confined particle must have kinetic energy, this energy increasing as the particle is more strongly confined. In the absence of a high pressure, the atoms cannot become sufficiently closely spaced to allow the formation of an ordered crystal, without the penalty of too large a zero point energy.

The Third Law of Thermodynamics requires that the entropy of a system in equilibrium should vanish at $T = 0$. Therefore the liquid must be in some sense completely ordered at $T = 0$. This ordering must be quantum mechanical in origin, as in the ordering of particles among quantum mechanical energy levels rather than in position. It seems reasonable to suppose that superfluidity is a consequence of this ordering.

3. THE HEAT CAPACITY

The heat capacity, C, is shown in Fig. 2, for the case when the helium is under its own vapour pressure. We see that the transition to superfluidity is accompanied by a large peak in the heat capacity. There is no latent heat, but the heat capacity tends to infinity at the transition, so that the transition cannot be classified as strictly second-order. The shape of the heat capacity near the transition is like a greek letter λ: hence the term $\lambda-$point to describe the transition. The type of anomaly depicted in Fig. 2 is

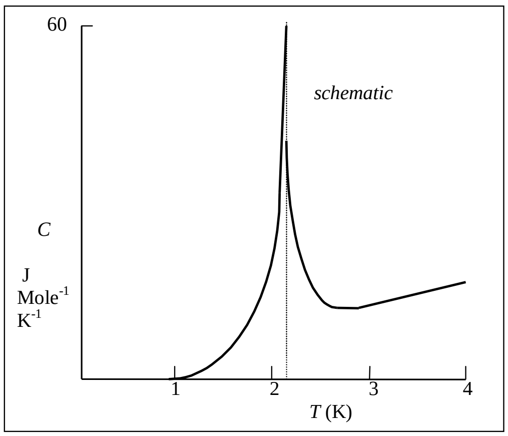

Fig. 2: The heat capacity of liquid ^{4}He.

quite common in nature, and it is characteristic of a system that exhibits an order-disorder transition; an example is the ferromagnetic transition. We see clear confirmation that superfluidity must be associated with a (quantum mechanical) ordering in the liquid. A similar anomaly in the heat capacity appears at the transition temperature of a superconductor, although in this case it has more nearly the character of a strictly second order transition.

We note that, although the heat capacity becomes rather small at low temperatures, it is quite large just below the $\lambda-$point; for example at 1.8K. This feature can be useful in applications.

4. THE OBSERVED PROPERTIES OF SUPERFLUID ^{4}He: THE TWO-FLUID MODEL

At first sight these properties present a confusing picture, but they make sense in terms of the two-fluid model, regarded as a purely phenomenological description. We describe the essential features of this model, and then give examples of properties that can be described in terms of it. The superfluid phase can be regarded as a mixture of two fluids, which can support different velocity fields. The **normal fluid**, with density ρ_n, flow velocity field $\mathbf{v}_n$ and conventional viscosity η_n, carries all the thermal energy and entropy in the system. The **superfluid component**, with density ρ_s and flow velocity field $\mathbf{v}_s$, can flow without friction and carries no thermal energy. The densities, ρ_n and ρ_s vary with temperature in the way shown in Fig. 3.

A pressure gradient will tend to drive both fluids in the same direction. An increase in temperature increases ρ_n but decreases ρ_s, so a temperature gradient tends to drive the superfluid component in one direction (towards to high temperature) and the normal fluid in the opposite direction.

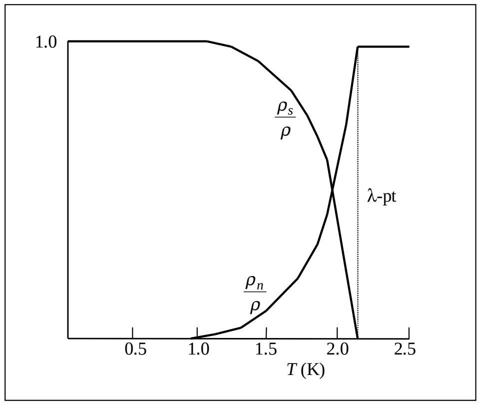

Fig. 3: The observed dependence of ρ_n and ρ_s on temperature.

5. EXAMPLES OF "TWO-FLUID" BEHAVIOUR

The superfluid component can flow without friction through even very narrow channels, so narrow that the normal fluid is rendered completely immobile by its viscosity. A striking example is provided by "film flow". Any solid surface in contact with the liquid is covered by a film of liquid, about 30 nm in thickness, as a result of van der Waals attraction between the helium atoms and the substrate. This is true in principle for any liquid, but in helium flow of the superfluid component through the very thin film becomes possible, with the result illustrated in Fig. 4.

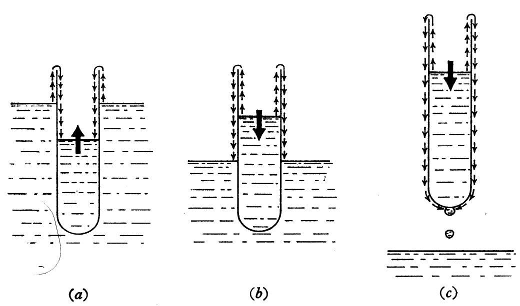

Fig. 4: Film fbw

A famous experiment was performed by Andronikashvili. He constructed a pile of discs, which he suspended in helium by a torsion fibre, as shown in Fig. 5. He measured the period of torsional oscillation as a function of temperature. The spacing between the discs was such that at the period of oscillation the normal fluid was completely coupled to the disc system. However, the superfluid component was not coupled, so that only the normal fluid contributed to the moment of inertia of the disc system. These measurements provided the first evidence for the dependence of normal fluid density on temperature shown in Fig. 3.

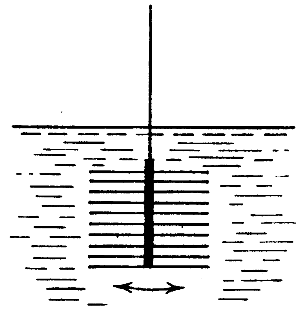

Fig. 5: The Andronikashvili experiment

Heat transport in superfluid helium takes place by counterflow of the two fluids, the superfluid component moving towards the source of heat and the normal fluid away from it, as shown in Fig. 6. Only the normal fluid carries thermal energy, at a rate per unit area, $Q = \rho S T v_n$, where S is the entropy of the helium per unit mass. This leads to very effective thermal transport, at a rate limited only by the small viscosity of the normal fluid. In practice the thermal transport is not quite as effective as is suggested by this idea, as will be explained later.

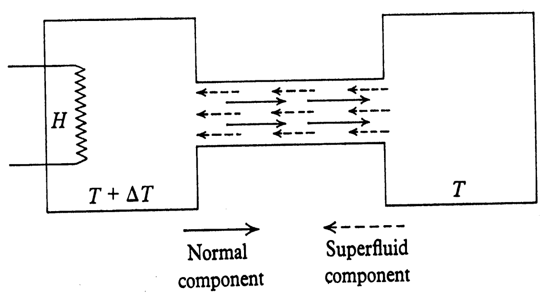

Fig. 6: Illustrating thermal transport by counterfow

The existence of two fluids allows two modes of longitudinal wave propagation. The two fluids can oscillate in phase, giving rise to **first sound**; or thay can oscillate in antiphase, giving rise to **second sound**. First sound is an isentropic pressure or density wave, analogous to ordinary sound in a fluid; it propagates at a speed of $c_1 = (\partial p/\partial \rho)_S^{1/2} \approx 240$ ms^{-1}. Second sound involves to a good approximation no change in density, but only a change in the proportions of the two fluids; it is therefore a temperature wave, but one that obeys the wave equation rather than the diffusion equation. The speed of second sound is given by $c_2^2 = TS^2\rho_s/C\rho_n$, and its value is roughly 20 ms^{-1} over the temperature range from 1K to 2K. Transient thermal effects in superfluid helium can therefore be very different from those in a conventional fluid, and discussion of them must allow for the existence of second sound.

The examples of two-fluid behaviour that we have described apply in their simplest form only if the flow velocities do not exceed certain critical values, which are often quite small ($\sim$few mm s^{-1}). We shall discuss the reason later, after we have explained the theoretical basis of the two-fluid model.

The two-fluid model applies also to superconductors. The resistive loss that occurs in a rapidly oscillating electric field is due to motion of the normal fluid.

6. WHY IS HELIUM DESCRIBED BY A TWO-FLUID MODEL?

Part of the answer to this question was given by Landau in his famous 1941 paper. In effect he focussed his attention on the nature of the normal fluid. He considered the form of the thermally excited states in the liquid at a low temperature. He argued with great insight (but less rigour) that they would consist of quantized sound waves, which are called **phonons**, and elementary forms of rotational motion called by him **rotons**. His ideas were placed on a firmer theoretical basis by Feynman, who was able to be more precise about the nature of a roton, which he showed to be in essence a free atom moving through the liquid, with a backflow formed from motion of the other atoms. Suppose that we set all these excitations into motion with a drift velocity $\mathbf{v}$, leaving the fluid otherwise at rest. Given the properties of the excitations (in particular their energy-momentum relationship, which can now be determined experimentally by neutron scattering) Landau calculated the momentum density, $\mathbf{J}_e$, associated with the drifting excitations. He found that

$$\mathbf{J}_e = \rho_e \mathbf{v} < \rho \mathbf{v}, \tag{1}$$

where the inequality holds at sufficiently low temperatures, which turn out to be temperatures below the $\lambda-$point. Thus the drifting excitations do not cause the whole fluid to drift, in the sense that they carry an effective density that is less than the total density of the helium.

We identify the gas of excitations with the normal fluid. Then $\rho_e = \rho_n$ can be calculated, and it can be shown to be equal to the observed normal-fluid density.

The superfluid component in Landau's picture is what is left over after the thermal excitations have been taken into account. Landau also considered what would happen if this background were to move. He showed that it could not slow up by creating or scattering excitations if its velocity were less that a critical value, which is about 60 ms^{-1}. This picture of the superfluid component is not wholly satisfying, and it is certainly not the whole story, not least because observed critical velocities are typically very much less than 60 ms^{-1}. We shall now examine the nature of the superfluid component in more detail, and we shall demonstrate its connection with Bose condensation.

7. THE NATURE OF THE SUPERFLUID COMPONENT

To understand the real nature of the superfluid component we must start by looking at the phenomenon of Bose-Einstein condensation. Bose condensation plays a crucial role in superfluidity, contrary to Landau's original opinion.

Consider an ideal gas formed from Bose particles: i.e. particles such as ^{4}He atoms that are quantum-mechanically indistinguishable, but are not subject to the exclusion principle (i.e. there can be any number of particles in one quantum state). If we calculate the way in which the particles of the gas are distributed over the quantum states determined by the shape and size of the containing vessel, we find an interesting result: below a critical temperature, T_0, a finite fraction of the particles are "condensed" into the lowest quantum state. The way in which this fraction varies with temperature is shown in Fig. 7(a), and the calculated heat capacity is shown in Fig. 7(b). The heat capacity reflects the ordering of the particles into a single quantum state below the temperature T_0. Very recently, such Bose condensation has been observed directly in weakly-interacting gases formed from alkali-metal atoms levitated magnetically and trapped in a vacuum, the gas being cooled below the temperature T_0 (typically in the range 0.1-1 μK) by a combination of laser and evaporative cooling [4].

For an ideal hypothetical gas of non-interacting helium atoms with the same density as liquid helium the condensation temperature $T_0 \sim 3$K. An obvious question is whether a similar type of ordering occurs in real liquid helium, albeit modified in some way by the strong interactions between the helium atoms.

The answer is that it does, as shown first by Penrose and Onsager. The fraction of condensed particles is smaller than in the ideal gas; even at $T = 0$ it is only about 10 percent. But it remains the

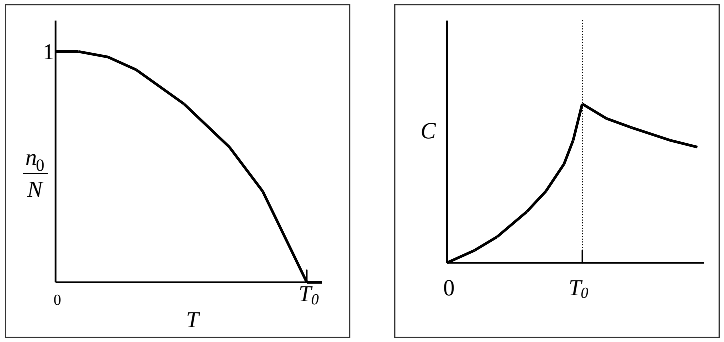

Fig. 7: The predicted behaviour of an ideal Bose gas. (a) The temperature dependence of the condensed fraction of particles; (b) the predicted heat capacity.

case that a macroscopic fraction, and a very large absolute number, of the atoms does condense into what is effectively a single quantum state, and it turns out that at $T = 0$ the non-condensed atoms are effectively locked to the condensed atoms.

We now understand that this is indeed the ordering process taking place below the $\lambda-$point, and that ultimately it is this ordering that is responsible for superfluidity. It is a remarkable process, because it is closely analogous to the formation of a coherent electromagnetic wave in a laser, which can be viewed as a condensation of photons into a single quantum state. In helium there is a coherent matter wave. A similar process occurs in a superconductor, except that the coherent wave is formed from Cooper pairs. A coherent matter wave lies at the heart of both superfluidity and superconductivity.

The assembly of condensed atoms is called the condensate, and the associated wavefunction is called the condensate wave function (CWF). If the condensed atoms are at rest the CWF is just a constant Ψ_0, where Ψ_0^2 is a measure of the number of condensed atoms. If they are moving, each with momentum $m_4 v$ along the x-axis, the CWF becomes

$$\Psi = \Psi_0 \exp\left(\frac{i m_4 v x}{\hbar}\right). \tag{2}$$

For a more general motion of the condensate we can write

$$\Psi = \Psi_0 \exp\left(iS(\mathbf{r})\right), \tag{3}$$

where the local velocity of the condensed atoms is equal to $(\hbar/m_4)\,\nabla S$. We identify this velocity with the velocity of the superfluid component

$$\mathbf{v}_s = \left(\frac{\hbar}{m_4}\right)\nabla S. \tag{4}$$

We can ask how this view of superfluidity relates to that proposed by Landau, which was very successful in accounting for two-fluid behaviour. We now know that the two approaches are intimately connected, in the sense that the *form* of the spectrum of the thermal excitations, which underlies Landau's calculation showing that $\rho_n/\rho < 1$ below the $\lambda-$point, is intimately connected with the existence of the condensate. Without the condensate the spectrum would have the wrong form. Note especially that we now have a clear view of the meaning of the velocity of the superfluid component, which was not provided by Landau.

A condensate exists also in a superconductor, formed from the Cooper pairs. The mass m_4 is replaced by $2m$, where m is the electron mass.

8. QUANTUM RESTRICTIONS ON SUPERFLUID FLOW

As we shall now demonstrate, the macroscopic occupation of a single quantum state in the Bose-condensed helium gives rise to macroscopic quantum effects, as London had foreseen.

It follows from Eq. (4) for the superfluid velocity that

$$\mathrm{curl}\mathbf{v}_s = 0. \tag{5}$$

This means that there can be no local rotational motion of the superfluid component. This is really a consequence of the quantization of angular momentum, as we see more clearly in a moment. But there *can* be a finite **hydrodynamic circulation**, defined as

$$\kappa = \oint_C \mathbf{v}_s \cdot d\mathbf{r}, \tag{6}$$

round any cicuit that cannot shrink to nothing while remaining in the fluid; for example, a circuit round a solid cylinder passing through the fluid (Fig. 8). However, the circulation cannot take any value. If we

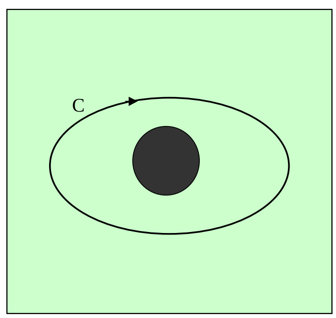

Fig. 8: Illustrating a circuit round which there can be a fi nite superfluid circulation.

substitute from Eq. (4) into Eq. (6) we obtain

$$\kappa = \frac{\hbar}{m_4} \oint_C \nabla S \cdot d\mathbf{r} = n\frac{2\pi\hbar}{m_4}, \tag{7}$$

where n must be an integer in order to satisfy the condition that the CWF be single-valued. This means that the **superfluid circulation must be quantized in units of** $2\pi\hbar/m_4$. This circulation is macroscopically large (it can be measured in a macroscopic mechanical experiment), and this fact provides the clearest evidence that superfluidity is indeed a “quantum mechanism on a macroscopic scale”. It arises from the quantization of angular momentum, combined with the fact that all the particles in the condensate must have the *same* angular momentum. In the absence of any quantized circulation there can be no local angular momentum, as we have seen in connection with Eq. (5). The quantization of circulation is has its analogue in superconductivity, where it is observed as the quantization of trapped flux.

9. WHY CAN THE SUPERFLUID FLOW WITHOUT FRICTION?

As we have mentioned, Landau showed that the flowing superfluid component cannot decay into excitations unless the velocity is very large. With the idea of the condensate we can gain greater insight into this frictionless flow.

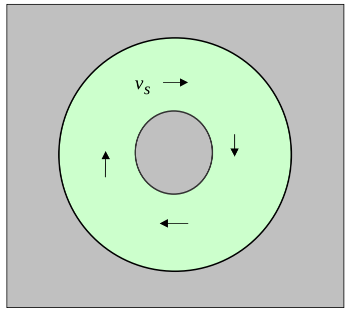

Fig. 9: Persistent superflow round a torus

Suppose that there is a persistent superflow round a torus, as shown in Fig. 9. This flow can be only metastable, because a state with no flow has a smaller (free) energy. Why is it metastable? The condensate contains a macroscopic number of atoms. Interaction of these atoms with the walls of the torus will cause scattering, and some atoms may as a result be knocked out of the condensate. This will reduce the amplitude of the CWF, but it will not alter its coherent phase. Therefore the superfluid velocity does not change, although the superfluid density may decay a little, which would correspond to the creation of more normal fluid in the form of excitations. Putting it in another way, we can say that the destruction of superflow would require a transition that takes a macroscopic number of atoms from one state to another simultaneously, and such a process has very low probability.

But superflow *can* decay through a mechanism that we have not yet considered: the creation of free vortex lines, to which we now turn our attention.

10. QUANTIZED VORTEX LINES IN SUPERFLUID HELIUM

We have seen that a quantized superfluid circulation can exist round a solid cylinder running through the helium. A free quantized vortex line in the superfluid component is a quantum of circulation round a tiny cylindrical hole in the helium. Such a line always has one quantum of circulation, and the hole then has a natural size, determined by a balance between the kinetic energy of flow and the surface energy of the hole, that is less than an interatomic spacing.

Such vortex lines can exist in superfluid helium, and, as we shall show, they play an important role in its behaviour. Most obviously, perhaps, they allow the superfluid component to rotate if the helium is placed in a rotating vessel; otherwise such rotation would be forbidden by Eq. (5). A parallel array of

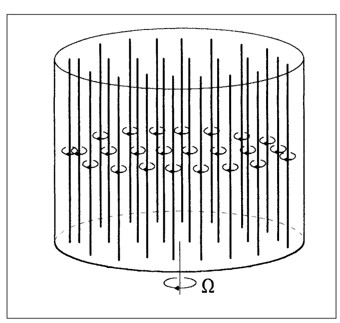

Fig. 10: Vortex lines in the uniformly rotating superfluid component.

lines, as shown in Fig. 10, gives rise to a flow field that mimics uniform rotation on length scales larger than the line spacing, which is about 0.2 mm at $\Omega = 1$ s^{-1}. This array is analogous to the array of flux lines in the mixed state of a type II superconductor.

Vortex lines scatter the excitations that constitute the normal fluid, and therefore they give rise to a frictional force between the two fluids, called **mutual friction**. This is observed as an attenuation of second sound when it propagates in the uniformly rotating helium. The observation of this attenuation provided the first experimental evidence for the existence of vortex lines.

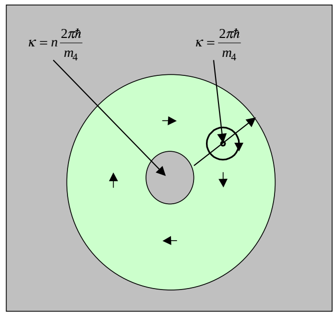

Fig. 11: Decay of a persistent current by vortex motion.

Vortex lines provide a new mechanism by which a persistent superflow can decay (Fig. 11). Consider again a persistent superflow in a torus. Let the persistent current consist of n quanta of circulation. If a free vortex, with the appropriate sign, crosses the channel, this value of n falls to $n - 1$. Does this mean that the current simply decays? It does not, because the movement of the free vortex across the channel is opposed by a potential barrier. This barrier arises because a vortex is attracted to a solid boundary by its image. The barrier is quite large in cases of practical interest, and it can be overcome only at high velocities ($>\sim$1-10 ms^{-1}), either thermally or by quantum tunnelling. **Without this barrier there would be no superflow.** The barrier exists only because a vortex has a finite quantized circulation, so it is quantum in origin. The barrier exists also in a superconductor, where it is usually called the Bean-Livingston barrier.

In practice frictionless superflow usually breaks down at velocities much less than 1 ms^{-1}. This is due to a few **remnant vortices**, which can expand and multiply, and then cross the channel (cf remnant dislocations in a solid allowing the solid to deform much more easily than might have been expected). Remnant vortices seem always to be created when the helium is cooled through the $\lambda-$point.

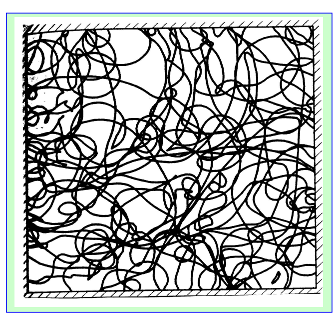

Fig. 12: A turbulent tangle of vortex lines.

This expansion and multiplication leads to a type of turbulence in the superfluid component: a kind of tangle of vortex lines (Fig. 12). Superfluid turbulence is very common. It seems always to be generated when the flow velocity exceeds a critical value that depends on channel size and is often as small as 1 mm s^{-1}.

11. PRACTICAL CONSEQUENCES OF SUPERFLUID TURBULENCE

Superfluid turbulence plays an important role in limiting heat transport in superfluid helium by counterflow. The counterflowing fluids cause remnant vortices to multiply (through the action of mutual friction), and this leads to a self-sustaining regime of homogeneous turbulence. The vortices thus generated lead to a steady average force of mutual friction per unit volume between the two fluids, given by

$$F_{sn} = A\rho_s\rho_n|v_s - v_n|^3, \tag{8}$$

which limits the heat transport rate, Q per unit area, in a way that is generally much more important than normal-fluid viscosity. The parameter A is about 800 m s kg^{-1} at 1.8K. Q becomes a non- linear function of the temperature gradient, which is given by

$$\nabla T = \frac{A\rho_n}{\rho_s^3 S^4 T^3} Q^3, \tag{9}$$

where S is again the entropy per unit mass of the helium. Although mutual friction becomes the dominant dissipative process limiting the heat flow, the effective thermal conductivity remains generally very high.

Superfluid helium can be forced to flow down a tube or past an obstacle, just as can any conventional fluid. Except at very small velocities or in very narrow channels both the superfluid component and the normal component become turbulent. It turns out that this turbulence is surprisingly similar to that in a conventional fluid at high Reynolds number. The reasons are complicated, but they seem to be connected with two facts: on a scale large compared with the spacing between the vortex lines even the superfluid component looks like a classical fluid flowing at high Reynolds number; and the mutual friction associated with the vortex lines serves to lock the two velocity fields together. Thus the flow of the superfluid phase of liquid helium at high velocities in situations having a classical analogue is described quite well by classical formulae describing the flow of a conventional fluid, with density equal to the total helium density and viscosity similar to that of the normal fluid (for a recent extensive review of quantum turbulence see reference [5]).

12. THE KAPITZA THERMAL BOUNDARY RESISTANCE

As we have seen the effective thermal conductivity of superfluid helium is very high, but often it is necessary to transfer heat out of a solid body into the helium, or *vice versa*. We must then take account of a high thermal boundary resistance between the solid and the helium (the Kapitza resistance). This resistance arises from the fact that it is generally difficult for a thermal excitation in the solid to convert to one in the helium. This can be seen most easily when both the excitations are quantized sound waves or phonons. When a sound wave approaches a change of medium, some is transmitted and some is reflected, the relative amounts being determined by the characteristic impedances ($Z = \rho c$) of the two media. For liquid helium Z has a value that is much smaller than for any solid, and the resulting serious acoustic mismatch at the boundary leads to the high thermal boundary resistance. Its value is typically of order 2×10^{-4} K W^{-1} m^2.

13. SUMMARY AND CONCLUSIONS

The superfluid phase of liquid ^{4}He behaves in strange ways, which can be summarized as follows. It shows "two-fluid" behaviour; a normal fluid coexisting with a superfluid component. The superfluid component can exhibit frictionless flow at low velocities and in narrow channels. Rotational motion in the superfluid component is severely restricted by quantum effects, associated with the quantization of circulation (essentially the quantization of angular momentum). This unconventional behaviour has its origin in quantum effects and especially in the formation of a coherent matter field within the liquid, associated with the phenomenon of Bose-Einstein condensation. At high flow velocities ideal superfluid behaviour, involving frictionless flow, breaks down through the generation of a form of quantum turbulence, which leads to a frictional interaction between the superfluid and normal components. Quantum turbulence is likely to be important in many situations of practical importance.

References

[1] D. R. Tilley and J. Tilley, *Superfluidity and Superconductivity*, 3rd ed. (Adam Hilger, Bristol, 1990).

[2] R. P Feynman, *Progress in Low Temperature Physics*, vol. 1, ch. 2, ed. C. J. Gorter (North Holland Publishing Co., Amsterdam, 1955).

[3] R. J. Donnelly, *Quantized Vortices in Helium II* (Cambridge University Press, 1991).

[4] See, e.g., *Bose-Einstein condensation in atomic gases*, *Proc. Int. School of Physics Enrico Fermi*, eds. M. Inguscio, S. Stringari and C. Wieman (IOS Press, Amsterdam, 1999).

[5] W. F. Vinen and J. J. Niemela, *J. Low Temperature Physics*, in press.