1. Consider a household refrigerator that uses HFC-134a as the refrigerant, operating between the pressures of 1.0 bar and 10 bars.
   a. The compression stage of the cycle begins with saturated vapor at 1 bar and ends at 10 bars. Assuming that the entropy is constant during compression, find the approximate temperature of the vapor after it is compressed.
   b. Determine the enthalpy of each of the points in the refrigeration cycle, and calculate the coefficient of performance (COP). Compare to the COP of a Carnot refrigerator operating between the same reservoir temperature. Does this temperature range seem reasonable for a household refrigerator? Explain briefly.
   c. What fraction of the liquid vaporizes during the throttling step?

2. In this problem, you will examine a useful thermodynamic potential for systems with variable contents.
   a. Perform a double Legendre transform of $U(S, V, N)$ to obtain a new thermodynamic potential $Φ(T, V, μ)$. This potential is called the Landau potential or the grand potential.
   b. Derive the fundamental thermodynamic relation for $Φ(T, V, μ)$ and the corresponding generalized forces for each natural coordinate.
   c. Determine the curvatures of $Φ(T, V, μ)$ along each axis. Determine the Maxwell relations associated with $Φ(T, V, μ)$.
   d. Prove that, for a system in thermal and diffusive equilibrium, $Φ$ tends to decrease.
   e. Based on the structure of $Φ(T, V, μ)$ [given in (a)-(c)] and its behavior as a thermodynamic system approach equilibrium [given in (d)], discuss the physical meaning of the grand potential.

3. One of the important uses of adiabatic compressibility $κ_s$ is the calculation of the speed of sound through a medium. Experimentation shows that sound transmission in a gas is an adiabatic process, primarily because sound waves cause pressure oscillations that are fast enough that heat cannot be transported by conduction from compressed regions to rarified regions. It can be shown (using fluid dynamics) that the speed of sound in a medium is $u^2 = (\partial P/\partial ρ)_S$.
   a. Using the equation of state in the form $ρ = ρ(T, P)$, where $ρ = 1/v$ and $v$ is the specific volume, show that the speed of sound is

$$u^2 = \left[ \frac{\partial ρ}{\partial P}_T - \left( \frac{κ_s}{κ_T} \right) \left( \frac{T}{C_v ρ^2} \right) \left( \frac{\partial ρ}{\partial T}_P \right)^2 \right]^{-1}$$

   b. Use the above expression to show that the speed of sound in an ideal gas is given by $u^2 = γRT$ where $R$ is the gas constant.
4. In this problem, you will examine some of the qualitative features of Helmholtz free energy.
   a. Suppose that you drop a brick on the ground and it lands with a thud. Explain why the energy of this system tends to spontaneously decrease. What happened to the total entropy of the universe?
   b. In previous lecture notes, we saw that the generalized forces for the Helmholtz energy were pressure and entropy. In particular, we noted that
   \[
   \left( \frac{\partial F}{\partial V} \right)_T = -P, \quad \left( \frac{\partial F}{\partial T} \right)_V = -S
   \]
   Explain why these formulas make intuitive sense by discussing graphs of \( F \) vs. \( V \) and \( F \) vs. \( T \) with different slopes.

5. In this problem, you will derive the chemical potential for an ideal gas as a function of internal energy and volume.
   a. For systems of variable composition, Gibbs equation can be written in terms of entropy.
      \[
      dS = \frac{dU}{T} + \frac{P}{T} dV - \frac{\mu}{T} dN
      \]
      Show that the two additional Maxwell conditions associated with entropy are given by
      \[
      T \left( \frac{\partial P}{\partial N} \right)_{U,V} - P \left( \frac{\partial T}{\partial N} \right)_{U,V} = -T \left( \frac{\partial \mu}{\partial V} \right)_{U,N} + \mu \left( \frac{\partial T}{\partial V} \right)_{U,N}
      \]
      \[
      \left( \frac{\partial T}{\partial N} \right)_{U,V} = T \left( \frac{\partial \mu}{\partial U} \right)_{V,N} - \mu \left( \frac{\partial T}{\partial U} \right)_{V,N}
      \]
   b. Using Equation (1) in part (a) and your knowledge of kinetic theory, show that the chemical potential for an ideal gas is
      \[
      \mu(U,V,N) = -\frac{2U}{fN} \ln \left( \frac{V}{V_0} \right) + \gamma(U,N)
      \]
      where \( \gamma(U,N) \) is yet to be determined and \( f \) is the number of degrees of freedom.
   c. Using Equation (2) in part (a) and your knowledge of kinetic theory, show that
      \[
      \gamma(U,N) = -\frac{U}{N} \ln \frac{U}{U_0} - U\lambda(N)
      \]
      where \( \lambda(N) \) is an arbitrary function of \( N \). Thus, the chemical potential for an ideal gas becomes
      \[
      \mu(U,V,N) = -\frac{2U}{fN} \left[ \frac{1}{2} \ln \left( \frac{U}{U_0} \right) + \ln \left( \frac{V}{V_0} \right) + \frac{fN}{2} \lambda(N) \right]
      \]
   d. Based on the above result, how does the 2\(^{nd}\) law restrain the flow of chemical potential?
**Bonus** [5 points]: Consider two isothermal expansions at temperatures that differ by \(dT\). Suppose we join the isotherms by isochoric processes at either end as shown below.

Let \(W\) and \(W'\) be the maximum amounts of work for processes \(AB\) and \(A'B'\), respectively. Show that the difference in the work done by the two isotherms can be written as

\[
\Delta W(T) = T \left[ \int \frac{\Delta U(T)}{T^2} dT + \alpha \right]
\]

where \(\alpha\) is a constant that is independent of temperature and where \(\Delta U = U_B - U_A\) is the difference in the internal energy between \(A\) and \(B\). Determine \(\Delta W\) for an ideal gas.