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Chapter 1: Preliminaries

1.1: What is Synoptic Meteorology?

Outside of field of meteorology, the adjective *synoptic* refers to a “summary or general view of a whole.” The adjective has a more restrictive meaning to meteorologists, however, in that it refers to large spatial scales. The first routinely available weather maps, produced in the late 19th century, were derived from observations made in European cities having a relatively coarse characteristic spacing. These early meteorological analyses, referred to as *synoptic maps*, paved the way for the Norwegian cyclone model, which was developed during and shortly after World War I. Because only extratropical cyclones and fronts could be resolved on the early synoptic maps, *synoptic* ultimately became a term that referred to large-scale atmospheric disturbances. Hence, *synoptic meteorology* traditionally involves the study of weather systems, such as extratropical high and low-pressure systems, jet streams and associated waves, and fronts. As we will see, the width of synoptic-scale features such as troughs and ridges in the midlatitude westerlies and large extratropical cyclones and anticyclones is much greater than their depth. Although fronts and jets associated with the aforementioned synoptic-scale features are as long as the troughs and ridges in the midlatitude westerlies, and have similar time scales, they are much narrower.

The study of weather phenomena on somewhat smaller spatial and temporal scales, *mesoscale meteorology*, includes the study of convective storms, land-sea breezes, gap winds, and mountain waves. In recent years, advances in observing and computing technology have allowed the boundary between synoptic and mesoscale meteorology to blur as an increasing volume of high-resolution information has become available. As the resolution of analyses, forecasts, and observations increases, mesoscale weather systems begin to come into focus on the maps that were once the exclusive realm of synoptic-scale weather systems. Today's synoptic meteorologist benefits from in-depth knowledge of mesoscale processes and their interactions with synoptic-scale weather systems. The earth system exhibits a continuous spatial and temporal spectrum of motion that defies simple categorization.

The question of whether to classify phenomena in terms of physical characteristics or controlling physical laws manifests itself in an interesting way in meteorology. Synoptic meteorology is largely based upon observation. On the other hand, dynamic meteorology is based on the acceptance of physical laws and deductions about atmospheric behavior based upon those laws. We can first observe a phenomenon and describe its characteristics, then analyze it to learn why it forms and why it behaves as it does, and ultimately to predict its behavior. Or we
can predict its existence based upon physical law, and then search for it in nature. Synoptic meteorologists usually take the former approach, whereas dynamic meteorologists usually take the latter approach. For example, the midlatitude cyclone was first observed, then analyzed, and much later numerically predicted. On the other hand, gravity waves were first discussed as solutions to a set of dynamical equations, and later were sought observationally. Today's synoptic meteorologist benefits from insights derived by physical laws (traditionally within the realm of dynamical meteorology) and the plethora of observational analysis based on our advances in observational technology and instrumentation.

Thus, knowledge of numerical modeling and modern observational systems, in addition to a solid traditional physics foundation in the atmospheric sciences, is consistent with increasingly high expectations for today's scientists to solve complex problems, often of considerable societal import. Atmospheric scientists who are able to synthesize theoretical concepts, observations, and conceptual and numerical models in their work are best able to contribute to scientific advance. The advantages of the interplay between practice and theory were recognized by pioneers in the atmospheric sciences, such as Vilhelm Bjerknes, who actively encouraged such diverse activities. Consider the following quote from C.-G. Rossby in 1934, a pioneer in the atmospheric sciences:

*The principle task of any meteorological institution of education and research must be to bridge the gap between the mathematician and practical man, that is, to make the weather man realize the value of a modest theoretical education, and to induce the theoretical man to take an occasional glance at the weather map.*

This quote embodies the spirit of atmospheric science and this philosophy remains as relevant today as it was in 1934.

Weather forecasting necessitates understanding a wide range of processes and phenomena acting on a variety of spatial and temporal scales. For example, forecasting for a coastal location requires information concerning the near-shore water temperature, the potential for land–sea breeze circulations, and the strength and orientation of the prevailing synoptic-scale wind flow. The prediction of precipitation type can benefit from knowledge of atmospheric thermodynamics and cloud physics. Other types of prediction, including air quality forecasting and seasonal climate prediction, also require knowledge that spans a broad spectrum of meteorological processes. Hence, weather forecasting and weather analysis are essentially topics in applied physics. In other words, the skill set necessary to become a proficient forecaster is the ability to synthesize information for a wide range of phenomena and to apply one's physics education (primarily classical mechanics, thermodynamics, and fluid dynamics) to understand and predict atmospheric phenomena.
1.2: Scales of Motion

As mentioned in the Preface, atmospheric motions occur over a broad continuum of space and time scales. The timescales of atmospheric motions range from under a second (in the case of small-scale turbulent motion), to as long as weeks (in the case of planetary waves). In order to examine atmospheric motions, we must identify the characteristic horizontal length and time scales, and these are often related to one another. The length scale can be related to the size of a weather system, whereas the time scale can be related to how long it would take an air parcel to circulate within the system.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Length (km)</th>
<th>Time</th>
<th>Example phenomena</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microscale</td>
<td>&lt;1</td>
<td>&lt;1 h</td>
<td>Turbulence, PBL</td>
</tr>
<tr>
<td>Mesoscale</td>
<td>1–1,000</td>
<td>1 h–1 day</td>
<td>Thunderstorm, land–sea breeze</td>
</tr>
<tr>
<td>Synoptic</td>
<td>1,000–6,000</td>
<td>1 day–1 week</td>
<td>Upper-level troughs, ridges</td>
</tr>
<tr>
<td>Planetary</td>
<td>&gt;6,000</td>
<td>&gt;1 week</td>
<td>Polar front jet stream, trade winds</td>
</tr>
</tbody>
</table>

The values in Table 1.1 are typically used to define different scale regimes in atmospheric systems. The values listed in Table 1.1 are approximate; there are not usually sharp distinctions or abrupt transitions in the dynamics at a certain specific scale, and there are exceptions. However, generally speaking, meteorological phenomena having short temporal scales tend to have small spatial scales, and vice versa.

1.3: Outline of the Book

No single course (or single textbook) can comprehensively treat the variety of processes and phenomena described previously, but the goal is to introduce these topics so that the reader develops an appreciation for the essential physics behind large-scale atmospheric phenomena. The approach of this course is to synthesize what has been traditional to synoptic meteorology (i.e., observations) with the theoretical foundation being laid by dynamical meteorology. The synoptic meteorologist represents the intuitive component of this process, while the dynamic meteorologist represents the deductive component of this process. The most important application of dynamics to synoptic meteorology, which we will examine in this course, is called quasigeostrophic theory, which is based upon observations, but logically developed by dynamical meteorology.
After introductory sections on units and coordinate systems, the kinematics of scalar meteorological fields, with an emphasis on the pressure and height fields will be discussed. The kinematics of the vector wind field is the next topic treated. We will then present an elementary survey of atmospheric dynamics and atmospheric thermodynamics with specific applications to synoptic fronts and other synoptic-scale motions. These discussions will ultimately culminate in the examination of quasigeostrophic theory. In the second portion of these notes, we will apply quasigeostrophic theory to the analysis of extratropical cyclones, anticyclones, upper-level waves, and the jet stream. Recognizing that the existence of mesoscale phenomena (such as thunderstorms, squall lines, etc.) depends upon the synoptic-scale environment and an understanding of convection, we will conclude the course with a discussion of atmospheric stability and convection.
Chapter 2: Observation of Meteorological Variables

The basic scalar meteorological variables we measure are pressure, temperature, humidity, and wind. The purpose of this chapter is to describe briefly the fundamental principles underlying common measurement instruments.

2.1: The Measurement of Time

The most basic meteorological variable is time. Weather systems and centralized produces regularly flow across time zones, so to help reduce confusion and simplify the workload, meteorologists preferentially use Universal Coordinated Time (UTC), also known as Greenwich Mean Time (GMT) or Zulu Time (Z). In simple terms, UTC is the time and date in London, England, ignoring any daylight saving time rules that are observed there. Forecasters should be thoroughly familiar with how to convert their local time zone to UTC and back. In the United States, Eastern Standard Time (EST) is converted by subtracting 5 hours from UTC time, or 4 hours when Eastern Daylight Time (EDT) is in effect. A map of time zones is given in Figure 2.1.
2.2: The Measurement of Pressure and Height

A very familiar and important meteorological property is pressure, which is defined as force per unit area. The derived SI unit of pressure is kg m$^{-1}$s$^{-2}$ = Pa, where Pa is the unit called Pascal. However, this unit of pressure is not used often in meteorology. The common meteorological convention is to measure pressure in millibars, which is defined as 1 mb = 1 hPa = 100 Pa.

Pressure may also be measured in terms of the height of a column of mercury in an evacuated tube, and in such a case is expressed in inches of mercury (note that one inch of mercury equals 33.86 mb). The volume of a liquid-like mercury changes as a function of pressure, and, when the liquid is confined inside a tube, the pressure may be inferred from the height of the liquid in the tube. If the volume of the liquid is also a weak function of temperature, then suitable corrections must be made. Ideally, the liquid should be chosen so that its volume is as sensitive to pressure, and as independent of other variables as possible.

Figure 2.2: (Left) Actual aneroid barometer. (Right) Aneroid barometer schematic

The volume of some materials also changes as a function of pressure. Barometers that work on this principle are called aneroid barometers; the extent to which the material bends is a function of pressure, and this bending action is converted into the deflection of a lever on a scale. As shown in Figure 2.2, an aneroid barometer is operated by a metal cell containing only a very small amount of air. Increased air pressure causes the sides of the cells to come closer together. On one side there is a fixed point to the base of the barometer, while the other is connected by levers and pulleys to a rotating pointer that moves over the scale. Aneroid barometers also are sensitive to temperature to some extent, and suitable corrections may have to be made.
The lever of some aneroid barometers is connected to a pen, which presses down upon a rotating cylinder containing a chart. Such a barometer is called a barograph, as shown in Figure 2.3. Good barometers are accurate to within 1 or 2 mb or better.

There are three primary pressure variable systems that are currently in use: station pressure (QFE), sea-level pressure (SLP), and altimeter setting (QNH). Station pressure is the actual, uncorrected pressure reading. A barometer that reads correctly at sea level and is brought to some other elevation, without adjustment, is displaying the station pressure. While it is not directly used in forecasting, it is the building block of all other pressure values. Sea-level pressure is that station pressure "reduced" numerically to sea level. The intent of this is to remove temperature bias from various stations at different elevations. The standard sea-level pressure is 1013.2 mb. Altimeter setting is station pressure value directly reduced to sea level. The altimeter setting is the value commonly heard on television and radio weathercasts in the United States and it is used for setting aircraft altimeter. The purpose of the altimeter setting is to remove elevation bias at various locations.

Height above the ground may be measured from an aircraft with a downward looking radar. Such a height is called the radar altitude. For a suitable environment (which will be demonstrated in a later chapter), knowledge of the pressure within the aircraft and the pressure at the observation station can be used to infer the height as a function of pressure and temperature. Heights are obtained that in general are good to within a meter or less in the lower depths of the troposphere.
2.3: The Measurement of Temperature

According to the kinetic theory of gases (which will be discussed in the next chapter), temperature is a measure of the amount of kinetic energy in the molecules of a substance, which manifests itself as how “hot” a substance is. The standard unit of temperature is given in degrees Kelvin (K) when computations are made. Temperature may also be measured in degrees Celsius (which is usually used on upper air weather charts) and in degrees Fahrenheit (which is usually used on surface weather charts in the United States). The temperature conversion formulas are made:

\[ T(\degree C) = \frac{5}{9} [T(\degree F) - 32], \quad T(\degree F) = \frac{9}{5} [T(\degree C) + 32], \quad T(K) = T(\degree C) + 273.15 \]

where \( \degree C \) is degrees Celsius and \( \degree F \) is degrees Fahrenheit.

Temperature, like pressure, may be measured from the amount of change in volume of a liquid such as mercury or alcohol confined inside a tube. It's very important to use a liquid that will not freeze if the temperature gets very cold. Temperature can also be measured electronically. The volume of some solid substances changes as a function of temperature, and hence the deflection of an electrically conductive substance can result in a change in the capacitance of a set of parallel plates. The electrical resistance of some materials changes as a function of temperature, and hence these devices are called thermistors. Temperatures can usually be measured to within 0.5 \( \degree C \) with such devices. Thermometers that are coupled to a pen and a moving chart are called thermographs.
An infrared or microwave radiometer (pointing upward if it is ground based and downward if it is airborne or on a satellite) can be used to obtain vertical profiles of the temperature field. Since the radiative properties of oxygen and water vapor in the infrared and microwave regions of the electromagnetic spectrum depend to a large extent on temperature, measurement of infrared and microwave radiation can be used to get an estimate of the vertical temperature profile. The infrared and microwave radiometers are passive devices in that they are remote sensors that do not radiate their own energy, but rather detect radiation from the atmosphere. These thermodynamic profiles are very useful in that they can make measurements at a fixed location nearly continuously in time.

2.4: The Measurement of Humidity

The Earth's air is made up of nitrogen, oxygen, argon, and other gases. The most important trace gas in Earth’s atmosphere for the sake of meteorology is the gaseous phase of water, known as water vapor. Because of its important in determining the distribution of precipitation and atmospheric stability, several different measurements for moisture are commonly used:

- **Mixing ratio**: The mixing ratio $q$ (units: g/kg) is simply the ratio of the mass of water vapor to the mass of dry air in a given volume. The greater the mixing ratio, the more water is contained in a given volume of air. On a summer day in Florida, a typical mixing ratio will be about 12 g/kg.
Relative humidity: Relative humidity is defined as the ratio of the mixing ratio $q$ to the saturation mixing ratio $q_s$. The saturation mixing ratio $q_s$ (units: g/kg) represents the maximum possible mixing ratio for a given parcel of air and it is largely a function of temperature. If $q = q_s$, then the relative humidity is 100%. At this point, the air is considered to be saturated, and water vapor must condense into liquid droplets or ice crystals.

Humidity is perhaps the most difficult variable to measure. The electrical resistance of some materials varies as a function of humidity. These sensors are used electronically. It is useful to use a material whose resistance responds as linearly as possible on humidity. The **sling psychrometer** is a set of thermometers mounted side by side. One wick is left dry, while the other is wet with pure water. The device is spun through the air so that the “wet-bulb” thermometer attains the wet-bulb temperature as the water evaporates, while the “dry-bulb” thermometer simply measures the air temperature. Infrared and microwave radiometers can be used to estimate the vertical profile of water vapor and liquid water in a manner similar to that which can be used to determine the vertical temperature profile.

![Sling Psychrometer Diagram](image)

Figure 2.6: (Left) An actual sling psychrometer; (Right) A schematic of a sling psychrometer

Clouds are best observed by satellite. Satellites in geosynchronous orbit (i.e. an orbit with an orbital period equal to the rotation period of the Earth) rotate about earth's axis at the same rate at which the Earth turns about its axis. Photographs of the same entire half of the Earth's disc are available at frequent intervals. Polar-orbiting satellites provide higher-resolution images because they are closer to Earth's surface. However, images of the same location are available only several times daily. Visible channels show clouds and surface features, while the infrared
channels show cloud-top and surface temperatures. Time-lapse movies or “loops” of images from a geosynchronous satellite are often used to view the evolution of cloud features.

2.5: The Measurement of the Wind Field

The wind field is the primary meteorological vector variable we measure, and is perhaps the most important. Meteorologists often express the wind speed (i.e. the magnitude of the velocity vector) in knots, one of which equals 0.514 m/s or 1.15 mph. The following conversion formulas are often useful.

- 1 meter = 39.37 inches (in.) = 3.2808 feet (ft)
- 1 kilometer (km) = 0.62137 statute miles
- 1 statute mile (mi) = 1.6093 km = 5280 ft
- 1 nautical mile (nm) = 1 min of latitude = 1.15 statute miles
- 1 m/s = 1.94 knots (kts) = 2.23 miles per hour (mph)
- 1° latitude = 111.137 km.

The measurement of wind is based upon two main principles:

1. The action of wind upon a fixed object produces a change in some properties of that object that depend upon the wind speed.
2. Some objects suspended in air move along (in the horizontal) at the same speed as the air and can be tracked.

Figure 2.7: (Left) An actual anemometer. (Right) Schematic of an anemometer
Devices in which the wind speed is related quantitatively to a change in some property of the device are called **anemometers**. The torque induced by the wind on a number of rotating cups is called a **cup anemometer**. Some anemometers work on the principle that the pressure change across the device depends upon the wind speed and direction. These anemometers have no moving parts. Sonic anemometers employ the principle that the sound waves travel at the air velocity plus the velocity of sound in still air. Anemometers are usually mounted on towers, high enough above surrounding obstacles so that the wind measurements are represent of the free air above the ground. Anemometers are often located at 10 m above ground level (also called **AGL**), the so-called **anemometer level**.

Meteorologists usually express this direction of the wind in degrees relative to true north, the direction of the North Pole. Meteorologists must be extremely attentive to whether a direction describes where wind is **blowing from** or **blowing to**. The **meteorological wind direction** is the compass heading from which the wind is blowing. For example, northerly, easterly, southerly, and westerly winds are from 360 (north), 90 (east), 180 (south), and 270 (west) degrees, respectively. In other words, the meteorological wind direction ($\theta_{\text{met}}$) is the angle made by the horizontal wind vector measured in a clockwise manner from the y-axis. However, **wind vectors** express which direction winds are blowing toward. In operational meteorology, wind vectors are used primarily when working out physical equations and when using the hodograph, while nearly all weather charts use wind direction instead. Fortunately, when it comes to actual weather charts, the use of wind symbols in meteorology is never ambiguous. Vector symbols are always reserved for wind vectors, whereas wind barbs are always an indicator of wind direction.

The wind may be estimated from the motion of objects that move along with the wind. There are many techniques for doing this. The most common method of this type involves the tracking of a balloon, which has a known ascent rate and which is released from the ground and visually tracked a theodolite (a precision instrument used for measuring angles in the horizontal and vertical planes). Balloons that are tracked by radar or by direction-finding devices (rotatable antennas that search for the strongest signal) are called **rawinsondes**. The accuracy of the standard rawinsonde varies from 1 m/s at low altitudes to 10-20 m/s near the tropopause around a jet. The accuracy diminishes when the balloon is far downstream at a low elevation angle.

Balloons may also be tracked with the use of navigation aids. Signals are received from several very low frequency (VLF) radio beacons around the world, and are re-transmitted downward to the fixed base station. The difference in time of arrival (measured by phase differences among the signals) of the radio beacons and a knowledge of the exact location of the beacons enables one to determine the precise location of the balloon. Winds may also be computed by determining the difference between the track taken by an aircraft as determined by an **inertial navigation system** (INS) aboard the aircraft and that expected by the orientation of
the aircraft. The INS determines the location of the aircraft from its initial position and measurements of the aircraft's acceleration as a function of time. The wind, therefore, blows the aircraft off course, and knowledge of this deviation enables one to determine the wind.

Cloud elements may be tracked in sequences of satellite photographs in order to get an estimate of the wind field at cloud-top level. This method is very useful over data-void regions such as those over the ocean. However, one must determine the height of the cloud top from an inspection of infrared photographs, and by associating cloud-top temperature with the temperature of a representative sounding. Finally, pulsed Doppler radar may be used to estimate the motion of targets such as precipitation and clouds.

There are a number of types of platforms upon which instruments are placed. Surface measurements of pressure and humidity may be measured by an observer reading the scale of a simple barometer and a sling psychrometer. Temperature may be read off a simple thermometer. Automated networks of observation stations (as shown in Figure 2.10) send digitally processed signals to a central computer. Aircrafts are also often used as platforms. In some cases the data may be relayed via satellite to fixed stations. Tall towers have been instrumented at various level so that profiles of temperature and humidity as a function of pressure may be obtained in the boundary layer (i.e. the layer closest to the Earth that is most affected by surface friction). Instruments placed under balloons that are released into the air and travel upwards and that
transmit data by radio back to the ground are called **radiosondes**. Instruments placed under parachutes that are then dropped from an aircraft are called **dropsondes**, as shown in Figure 2.9. Finally, satellites are used as bases for the remote measurement of temperature and humidity in the atmosphere below.

### 2.6: Observation Stations

Weather observations are taken daily every 1 to 6 hours at about 5,000 stations around the world and shared via data networks under the oversight of the World Meteorological Organization (WMO) and International Civil Aviation Organization (ICAO). Because of the close relationship of meteorology with aviation, a substantial number of weather stations are located at airports and are managed by aviation authorities. This is especially true in the United States, Canada, and Europe.
The United States uses 3 station identifiers: ICAO location indicators, FAA location identifiers, and WMO station index. The ICAO codes were developed in the mid-1950s by the International Civil Aviation Organization, which standardizes and publishes the codes. The first one or two letters always represents the country or region (for example, a U.S. identifier is K---). The FAA codes were developed in the 1940s by the Civil Aeronautics Administration. In the U.S., where ICAO identifiers begin with K, FAA codes are generally equivalent to the second, third, and fourth letters. The WMO codes were developed as 5-digit numerical identifiers in 1948 and are widely used within meteorology. These codes were developed to resolve growing chaos caused by dozens of station code systems and weather dissemination formats that varied from country to country. Much like the ICAO system, the first two digits identify the country or region and are known as block numbers (for example, the U.S. has a block number of 72). Meteorologists frequently come into contact with these codes when dealing with soundings, radiosonde observations, and synoptic observations.

METAR is the most common format for hourly surface observations in North American and Europe. At some stations, a METAR observation is transmitted every 20 minutes, and if a significant weather change occurs it is referred to as a SPECI (special) observation. The format is rather readable and is designed mostly for the aviation sector. The Federal Meteorological Handbook (http://www.ofcm.gov/fmh1/fmh1.htm) on surface weather observations contains a full list of abbreviations for METAR reports.

As mentioned previously, radiosondes are the most important (and most reliable) source of weather data for the upper atmosphere. Twice a day, hundreds of radiosonde stations around world launch these instruments and monitor the primary meteorological variable. These are encoded in a code format known as WMO TEMP (or RAOB) and are disseminated worldwide, where they can be used in soundings and integrated into model forecasts. The Federal Meteorological Handbook (http://www.ofcm.gov/fmh3/pdf/13-app-e.pdf) on rawinsonde observations contains a full list of abbreviations for radiosonde codes.
2.7: Weather Data Visualization

After data is collected at various weather stations, the observational data is processed and presented in many forms. The most common forms of weather data visualization are: surface and upper-air weather maps; thermodynamic diagrams; numerical model output; satellite imagery; and radar imagery. These weather charts are openly available to the public through the internet. Below is a list of popular institutions that visualize real-time weather data:

- Plymouth State Weather Center: [http://vortex.plymouth.edu/index.html](http://vortex.plymouth.edu/index.html)
- University of Oklahoma: [http://hoot.metr.ou.edu/](http://hoot.metr.ou.edu/)
- Pennsylvania State University Electronic Map Wall: [http://mp1.met.psu.edu/~fxg1/ewall.html](http://mp1.met.psu.edu/~fxg1/ewall.html)

In the next chapter, we will examine the kinematics of scalar meteorological fields and the vector wind field.
Chapter 3: Kinematics of Meteorological Variables

The basic meteorological variables are pressure, temperature, humidity, and the wind fields. The purpose of this chapter is to briefly describe the kinematics of these meteorological fields. We first consider pressure (at constant elevation) because it is the most commonly analyzed quantity.

3.1: The Scalar Meteorological Fields

![Sea-level pressure map of the continental U.S. on 26 March 2014 at 17 Z](image)

A sea-level pressure map of the continental United States given in Figure 3.1. Typically on surface maps, lines of constant pressure (which is usually at a fixed elevation) are plotted. These lines are called isobars. Suppose we regard a curved isobar as an arc section of an imaginary circular isobar. The radius of curvature vector $\vec{R}$ is measured from the center, radially outward to the isobar. If pressure increases in the radially outward direction, then the isobar is considered to be positively curved. Conversely, if pressure decreases in the radially
outward direction, then the isobar is considered to be **negatively curved**. In the former case, pressure is an increasing function of distance from the center, and the radius of curvature is positive (such as the low-pressure center over Colorado in Figure 3.1). In the latter case, pressure is a decreasing function of distance from the center, and the radius of curvature is negative (such as the high-pressure center over Tennessee over Colorado in Figure 3.1). An isobar is **negatively curved** if the isobar is concave in the direction of increasing values of pressure. If an isobar is convex in the direction of increasing values of pressure, then the isobars are said to be **positively curved**.

![Figure 3.2 Annotated surface analysis chart for 18Z on 04 April 2011. Courtesy of NOAA. Trough axes are denoted by the dashed lines.](image)

A **ridge axis** (or **ridge line**) may be defined as the locus of maximum negative curvature on a set of adjacent isobars. A **trough axis** (or **trough line**) may be defined as the locus of maximum positive curvature on a set of adjacent isobars. The symbol for a ridge axis is a sawtooth-shaped line, and the symbol for a trough axis is a dashed line. As shown in the surface analysis of Figure 3.2, trough axes are drawn over the low pressure trough in the Northwest US and over the Great Lakes near the Canadian border. Trough axes and ridge axes may have any orientation. **Tilted** troughs and ridges are those whose axes are not oriented strictly in the north-south direction. For example, the axes of positively (negatively) tilted troughs lean toward the east (west) with increasing latitude. In Figure 3.2, we see a positively tilted trough over the Northwest US, and a negatively tilted trough over the Great Lakes. As we will show later,
negatively tilted troughs are usually indicators of potentially severe weather, whereas positively tilted troughs often are signs of a weakening weather system.

Alternatively, one may think of positively and negatively tilted troughs and ridges as those that lean in the direction of the basic flow or against the direction of the basic flow, respectively, with increasing latitude. Troughs in easterly flow are found, for example, in coastal fronts along the east coast of the United States, in thermal troughs along the west coast of the United States, and north of developing cyclones in the central United States. (Since most of the pressure troughs we observe are in the midlatitude westerlies, troughs in easterly flow are often called inverted troughs. This terminology is misleading since the definition of a trough is independent of its orientation). Zonally oriented troughs are often found along zonally oriented warm fronts and stationary fronts (such as the trough axis over the Great Lakes). On surface maps, the relative strength of troughs and ridges are governed by the spacing between the isobars. Thus, strong ridges (troughs) have more tightly spaced isobars than weak ridges (ridges).

Features in the pressure field such as highs, lows, troughs, and ridges usually exhibit some motion. We often attribute motion to low- and high-pressure centers and to troughs and ridges, as if the lows and highs and troughs and ridges were solid bodies embedded within the flow of the atmosphere. In fact, the motion of lows, highs, troughs, and ridges is usually only “apparent”. These features are “propagating” when they re-form on one side and dissipate on the other. Air is continually being circulated into and out of these pressure-field features. In a sense, air is ingested, processed, and expelled from these features, just as air enters the human body, is processed, and is expelled in a transformed state. Thus, lows, highs, troughs, and ridges cannot be considered by themselves in isolated from their environment. The behavior of the pressure systems depends to some extent upon the trajectories of air parcels entering the systems. The injection of warm or moist or cold or dry air can be dynamically important.

Systems that move eastward or westward move zonally, while those that move northward or southward move meridionally. Troughs in the midlatitude westerlies that have a component of motion toward the equator are said to be digging. Troughs that have component of motion toward the pole are said to be lifting out. Pressure systems not only move, but they also intensify or weaken, or just have central pressures that change without any corresponding change in intensity. When the pressure in a high or along a ridge rises, it is said that the high or ridge is building. When the pressure falls in a high or along a ridge it is said that the high or ridge is weakening. When the pressure in a low or along a trough falls, it is said that the low or trough is deepening. Intensification occurs only if the spacing between isobars decreases. When the pressure rises in a low or along a trough it is said that the low or trough is filling. It is likely, but not always true, that deepening lows and troughs are also intensifying, and it is also likely, but not always true, that building highs and ridges are intensifying, and vice versa.
The rate at which intensity of the pressure field changes is called the **pressure tendency** and lines of constant pressure tendency are called **isallobars**. An isoballaric tendency map is given in Figure 3.3. The positive isallobaric tendency over Mississippi indicates that the pressure is rising locally. The rise in pressure could mean that a ridge axis or a high-pressure center has a component of motion that is toward the observer, or that a trough axis or a low-pressure center has a component of motion away from the observer, or that a nearby high-pressure area or ridge is stationary, and building, or that a nearby low-pressure area or trough is stationary and filling, or a combination of all the aforementioned. The pressure tendency field is important because it provides a mean of predicting the movement of surface pressure systems. Generally speaking, **lows tend to move from an adjacent region of greatest pressure rises toward an adjacent region of greatest pressure falls; highs tend to move from an adjacent region of greatest pressure falls toward a region of greatest pressure rises**. Based on this rule, Figure 3.3 suggests that frontal system along the eastern US will tend to move in an
eastward or east-northeastward direction. The exact direction of motion depends also on the intensity and symmetry of the pressure field.

A surface temperature map of the continental United States is given in Figure 3.4. The discussion on the kinematics of the pressure field is also valid for other scalar fields such as the temperature field and the moisture field. A line of constant temperature is called an isotherm and an isallotherm is a line of constant temperature tendency. There is some additional terminology often used to describe features in the temperature and moisture fields. A ridge in the temperature field is sometimes called a warm tongue. A trough in the temperature field is sometimes called a cold tongue. A local minimum in the temperature field is sometimes referred to as a cold pool.
A surface dewpoint map of the continental United States is given in Figure 3.5. A line of constant humidity is called an isohume. A ridge in the water-vapor field is often called a moist tongue or moist axis, while a trough in the water-vapor field is sometimes called a dry tongue or dry slot.

3.2: Advection

The atmosphere is a continuously evolving medium and so the fundamental variables discussed in the previous section is ceaselessly subject to temporal changes. But what does it really mean to say “The temperature has changed in the last hour”? In the broadest sense this statement could have two meanings. It could mean that the temperature of an individual air parcel, moving past a thermometer, is changing as it migrates through space. In this case, we would be considering the change in temperature experienced while moving with a parcel of air. However, the statement could also mean that the temperature of the air parcels currently in contact with the thermometer is lower than that of air parcels that used to reside there but have since been replaced by the importation of these colder ones. In this case we would be considering
the changes in temperature as measured at a fixed geographic point. These two notions of
temporal change are clearly not the same, but one might wonder if and how they are physically
related. We will consider an example to illustrate this relationship.

Imagine a winter day in Madison, Wisconsin characterized by biting northwesterly winds
which are importing cold arctic air southward out of central Canada. From the fixed geographical
point of my back porch, the temperature drops with the passage of time. If, however, I could ride
along with the flow of the air, I would likely find that the temperature does not change over the
passage of time. In other words, a parcel with $T = -3^\circ F$ passing my porch at 8 am still has
$T = -3^\circ F$ at 2 pm even though it has traveled nearly to Chicago, Illinois by that time. Therefore,
the steady drop in temperature I observe at my porch is a result of the continuous importation of
colder air parcels from Canada. We can write an expression for this relationship we’ve
developed:

\[
\text{Change with Time Following an Air Parcel} = \text{Change with Time at a Fixed Location} - \text{Rate of Importation of Temperature by Movement of Air}
\]

The left hand side of our expression is called the **Lagrangian rate of change** and it physically
describes how the temperature of an air parcel changes during its motion. The first term on the
right hand side of our expression is called the **Eulerian rate of change** and it physically
describes how the temperature changes at a fixed location over time. The second term on the
right hand side is called **advection** and it physically describes the rate of importation by the flow.

Advection is one of the most important processes in synoptic meteorology. Based on our
above example, there are three factors that govern the magnitude of advection:

(i) magnitude of the wind speed,
(ii) the spacing of the isotherms, and
(iii) the orientation of the wind vector with respect to the isotherms.

In particular, in order to maximize the magnitude of advection, there should be a strong wind
field that blows perpendicular to tightly spaced isotherms (see Figure 3.6). Conversely, no
advection occurs if the winds are parallel to isotherms. These considerations suggest the
following mathematical relationship for the advection of a scalar meteorological variable $\psi$

\[
|ADV(\psi)| = |\vec{V}| |\nabla\psi| \cos \theta
\]

where $|\vec{V}|$ is the magnitude of the wind field (i.e. the wind speed). $\nabla\psi$ is defined as the **gradient**
of the scale meteorological variable $\psi$. Physically, $\nabla\psi$ us a vector that describes the rate of
change of the function $\psi$, i.e., $\nabla\psi$ is the derivative of variable $\psi$. Mathematically, the direction
of $\nabla \psi$ points in the direction of maximum increase of $\psi$ and the magnitude of $\nabla \psi$ gives the slope of $\psi$ in the direction of maximum increase.

Consider the schematic diagram of warm and cold advection shown in Figure 3.6. Since the gradient vector points towards higher values of temperatures, this implies that $\nabla T$ points in the northerly direction. Moreover, note that $\nabla T$ always points perpendicular to isotherms. This is a general property of gradient vector and thus, it will apply to the gradient of any scalar variable. In the case of no temperature advection, $\vec{V}$ points perpendicular to $\nabla T$. In the case of cold (warm) advection, $\vec{V}$ points in the same (opposite) direction as $\nabla T$. Physically, this states that cold (warm) advection involves the transport of air parcel across cold (warm) isotherms to warm (cold) isotherms.

The above discussion can be generalized to discuss the advection of any scalar quantity. When the scalar quantity represents temperature, a positive (negative) value of advection is called warm advection (cold advection). When the scalar quantity represents water vapor, a positive (negative) value of advection is termed moisture advection (dry advection).
**3.2.1: Mathematical Derivation of Advection**

The relationship between the Lagrangian rate of change and Eulerian rate of change can be made mathematically rigorous. Doing so will assist us later in the development of the equations of motion that govern the mid-latitude atmosphere. Let’s consider an arbitrary scalar (or vector) quantity that we will call \( Q \). If \( Q \) is a function of space and time, then \( Q = Q(x,y,z,t) \). From differential calculus, the total differential of \( Q \) is given by

\[
dQ = \frac{\partial Q}{\partial x} \, dx + \frac{\partial Q}{\partial y} \, dy + \frac{\partial Q}{\partial z} \, dz + \frac{\partial Q}{\partial t} \, dt
\]

where the subscripts refer to the independent variables that are held constant while taking the indicated partial derivatives. Dividing both sides by \( dt \), the total differential of \( t \) which represents a time increment, the resulting expression is

\[
\frac{dQ}{dt} = \left( \frac{\partial Q}{\partial x} \right)_{y,z,t} \frac{dx}{dt} + \left( \frac{\partial Q}{\partial y} \right)_{x,z,t} \frac{dy}{dt} + \left( \frac{\partial Q}{\partial z} \right)_{x,y,t} \frac{dz}{dt} + \left( \frac{\partial Q}{\partial t} \right)_{x,y,z} \frac{dt}{dt}
\]

The rates of change of \( x, y, \) or \( z \) with respect to time are simply the components of the velocity vectors. Let’s define these components as \( u = dx/dt \), \( v = dy/dt \), and \( w = dz/dt \), respectively. Substituting these expressions gives

\[
\frac{dQ}{dt} = \left( \frac{\partial Q}{\partial x} \right)_{y,z,t} + u \left( \frac{\partial Q}{\partial x} \right)_{y,z,t} + v \left( \frac{\partial Q}{\partial y} \right)_{x,z,t} + w \left( \frac{\partial Q}{\partial z} \right)_{x,y,t}
\]

This can be written in vector notation as

\[
\frac{dQ}{dt} = \left( \frac{\partial Q}{\partial t} \right) + \vec{V} \cdot \nabla Q \Rightarrow \left( \frac{\partial Q}{\partial t} \right) = \frac{dQ}{dt} - \vec{V} \cdot \nabla Q
\]

We see that \( dQ/dt \) corresponds to the Lagrangian rate of change, \( \partial Q/\partial t \) corresponds to the Eulerian rate of change, and the rate of importation by the flow is represented by \(-\vec{V} \cdot \nabla Q\). Thus, \(-\vec{V} \cdot \nabla Q\) will be referred to as advection of \( Q \). The minus sign indicates that advection is considered positive when the rate of import in directed from positive values of \( Q \) to negative values of \( Q \). Thus, the Eulerian (fixed location) change is equal to the sum of the Lagrangian (parcel following) change and advection.

In the previous example, we imagined a temperature drop at my back porch. We also surmised that the temperature of individual air parcels did not undergo any change as the day wore on. Thus, the advective change at the porch must be negative – there must be negative
temperature, or cold air advection (i.e. \(-\vec{V} \cdot \nabla Q < 0\)), occurring in Madison on this day. Clearly, the situation of northwesterly winds importing cold air southward out of Canada fits the bill.

3.3: Overview of Atmospheric Thermodynamics

The relationships between the scalar meteorological variables that we discussed previously are part of the general field of physics known as thermodynamics. Thermodynamics is a branch of physics concerned with heat and temperature and their relation to energy and work. A quantitative description of thermal phenomena requires a careful definition of such important terms as temperature, heat, and internal energy. Moreover, thermodynamics states that the behavior of these variables is subject to general constraints that are common to all materials and these constraints are expressed in the laws of thermodynamics. In this section, we will discuss the properties of these variables that are most relevant to synoptic meteorology.

3.3.1: Macroscopic Description of Moist Air

Here, we examine the properties of a gas of mass \( m \) confined to a container of volume \( V \) at a pressure \( P \) and a temperature \( T \). It is useful to know how these quantities are related. In general, the equation that interrelates these quantities, called the equation of state, is very complicated. However, if the gas is maintained at a low density, the equation of state is quite simple and can be found experimentally. Such a low-density gas is commonly referred to as an ideal gas. In our atmosphere, air matches the description of an ideal gas. For our purposes, we can separate air into two basic components: dry air components (such as nitrogen, oxygen, etc.) and water vapor. The equation of state for the dry air component can be written as

\[
P_d = \rho_d R_d T
\]

where \( P_d \) is the pressure associated with dry air, \( \rho_d \) is the density of dry air, and \( R_d = 287 \, J \, K^{-1} \, kg^{-1} \) is known as the dry air constant. The equation of state for water vapor can be written as

\[
e = \rho_v R_v T
\]

where \( e \) is the pressure associated with water vapor (also known as vapor pressure), \( \rho_v \) is the density of water vapor, and \( R_v = 461 \, J \, K^{-1} \, kg^{-1} \) is water vapor gas constant.

It’s important to note that \( R_d < R_m \Rightarrow \rho_v < \rho_d \), which leads to common observational fact that moist air is less dense than dry air (assuming that the temperature is the same). This also means that the amount of water vapor in the air affects the density of air within the atmosphere.
How can we write the equation of state for moist air? This can be derived by noting that the density of moist air can be written as

$$\rho = \rho_d + \rho_v = \frac{P_d}{R_dT} + \frac{e}{R_vT} = \frac{P_d}{R_dT} + \frac{e}{R_vT}$$

Now, the total pressure exerted by moist air is $P = P_d + e$. Substituting this into the above equation gives

$$\rho = \frac{P - e}{R_dT} + \frac{e}{R_vT} = \frac{P}{R_dT} - \frac{e}{R_dT} + \frac{e}{R_vT} = \frac{P}{R_dT} \left[1 - \frac{e}{P} + \frac{e(R_d)}{R_vT}\right]$$

Rearranging this expression gives

$$P = \rho R_d T \left[\frac{1}{1 - \frac{e}{P} \left(1 - \frac{R_d}{R_v}\right)}\right]$$

The term in the bracket is a correction term that accounts for the amount of water vapor in the air. Now, we can define a new variable known as the virtual temperature $T_v$, which is given by

$$T_v = T \left[1 - \frac{e}{P} \left(1 - \frac{R_d}{R_v}\right)\right]^{-1}$$

Thus, the equation of state for moist air can be written as $P = \rho R_d T_v$.

Virtual temperature $T_v$ is an expression of temperature that takes into account the density of water vapor. Water vapor is over a third less dense than dry air, so adding it to dry air reduces its density. Virtual temperature is simply the temperature of a hypothetical mass of dry air whose density equals that of the sample containing water vapor. It can be shown that the virtual temperature can be approximated with the formula is $T_v \approx T + q/6$, where $T$ is measured in degrees Celsius or Kelvin and $q$ is the water vapor mixing ratio. It can be seen that by increasing the moisture, $T_v$ will be as much as several degrees higher than the ambient air temperature. This increases the buoyancy of air.

Another common variable used to describe moist air is the dewpoint temperature. Dewpoint temperature $T_d$ is the temperature at which saturation will occur if the air is cooled, omitting any change in pressure. When this point is reached, water vapor will condense, which leads to clouds and precipitation in the atmosphere. Dewpoint temperature is proportional to the amount of water vapor in the air, so it makes a good indicator of absolute (actual) moisture.
Since the dewpoint temperature is usually compared to the actual air temperature, another variable used to provide a measure of moisture is **dewpoint depression**. Dewpoint depression $T_{dd}$ is simply the difference between the actual air temperature and the dewpoint temperature, $T_{dd} = T - T_d$. The lower the value, the closer the air is to saturation and the higher the relative humidity. A dewpoint depression of less than 5°C at a given level in the atmosphere is considered to be suitable for the formation of clouds.

### 3.3.2: Kinetic Theory of Gases

In the previous subsection, we discussed the properties of an ideal gas using macroscopic, measurable variables such as pressure, volume, and temperature. We shall now show that such large-scale properties can be described on a microscopic scale. This theory will provide us with a physical basis for our understanding of the concept of temperature.

![Figure 3.7 Schematic of gas contained with a container](image)

Just as before, we examine the properties of a gas of mass $m$ confined to a container of volume $V$ at a pressure $P$ and a temperature $T$. For the properties of air in our atmosphere, we can also add the following additional assumptions:

- All molecules in the gas are identical. This means that gas under consideration is a pure substance, rather than a heterogeneous mixture.
- The molecules interact only through short-range forces during elastic collisions. This means that, in the collisions, both kinetic energy and momentum are constant.
- The molecules obey Newton’s laws of motion
The number of molecules in the gas is large and the average separation between molecules is larger compared with their dimensions. This means that the volume of the molecules is negligible compared with the volume of the container.

Consider a one-dimensional gas in a one-dimensional box of length $L$. The change in momentum after the molecule collides with the wall is

$$\Delta p_x = 2mv_x$$

Since the molecule must travel a distance $2L$ before returning to the same wall, the rate at which the molecules imparts momentum to the wall is given by Newton’s 2nd law:

$$F_{mol} = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

If there are $N$ molecules in the box, then the average force on the wall is

$$\langle F \rangle = \frac{Nm\langle v_x^2 \rangle}{L}$$

The pressure on the wall is given by

$$P = \frac{\langle F \rangle}{A} = \frac{Nm\langle v_x^2 \rangle}{LA} \Rightarrow PV = Nm\langle v_x^2 \rangle$$

Since the molecules are equally probable to move in all three directions of space, then we have

$$PV = \frac{1}{3}Nm\langle v^2 \rangle \Rightarrow P = \frac{2}{3}\left(\frac{N}{V}\right)\left(\frac{1}{2}m\langle v^2 \rangle\right) = \frac{2}{3}\left(\frac{N}{V}\right)\langle KE \rangle$$

This result indicates that the pressure is proportional to the number of molecules per unit volume and the average kinetic energy of the molecules. Since pressure is proportional to temperature, this means that temperature is a direct measure of the average molecular kinetic energy of the gas. Therefore, according to the kinetic theory of gases, temperature is an expression of the amount of kinetic energy in the molecules of a substance. In meteorology, this manifests itself as heat.
3.3.3: The First Law of Thermodynamics

One of the fundamental laws of physics is the conservation of energy, which asserts that the total energy of the universe remains constant. This principle can be applied to examine macroscopic systems in the atmosphere. The first law of thermodynamics is essentially a law of conservation of energy for macroscopic systems. Conservation can be imposed by requiring that the variation of the total energy of the system and its environment is identically zero. However, in order to make this precise, one should be able to say what exactly the energy of a macroscopic system is in the first place.

At the outset, it is important that we make a major distinction between internal energy and heat. Internal energy is all the energy of a system that is associated with its microscopic components (i.e. atoms and molecules). Thus, the internal energy includes translational kinetic energy, rotational kinetic energy of the molecules, the potential energy within molecules, and the potential energy between the molecules. Heat is defined as the transfer of energy across the boundary of a system due to temperature differences between the system and its surroundings. When you heat a substance, you are transferring energy into it by placing it in contact with surroundings that have a higher temperature.

The first law of thermodynamics says that a change in internal energy is due to

- **Heat** $Q$: Energy flow between a system and its environment due to a temperature gradient across a boundary and a finite thermal conductivity of the boundary and/or;
- **Work** $W$: Mechanical energy (and any other kind of energy) transfer across the boundary.

This indicates that work and heat are both defined to describe energy transfer across a system boundary. This law can be written mathematically as

$$dU = dQ + dW$$

where $dU$ is the differential change in internal energy, $dQ$ is the transfer of energy via heat, and $dW$ is the differential work done on the system.

In the atmosphere, the system is usually specified as an air parcel (which is an infinitesimal amount of air) and the environment is the environmental air surrounding the air parcel. If some external forces are acting upon the air by virtue of its interaction with the environment then there will be a change in the internal energy. If the system is an open system, then energy will be exchanged between the system and the environment in the form of heat. This implies that the change in internal energy should be equal to the work done on the system plus the heat added into the system. For the atmosphere, the work done on the system primarily occurs due to change in the volume of the air parcel. In particular, energy enters the air parcel
when work is done to compress the air parcel. Thus, the work can be calculated by using the common definition of work

\[ dW = F \, dr = (PA) \, dr = -P \, dV \]

where the minus sign indicates that compression of the parcel increases the parcel’s energy. Therefore, the first law can be written as

\[ dU = dQ - P \, dV \]

Increases in internal energy in the form of molecular motions are manifested as increases in temperature. This was experimentally verified by English physicist James Joule (1818-1889). Joule showed following a series of lab experiments that when a gas expands into a vacuum without doing external work \((dW = 0)\) and without taking in or giving out heat \((dQ = 0)\), that the temperature of the gas does not change. By the first law of thermodynamics, \(dW = 0\) and \(dQ = 0\) implies that \(dU = 0\). This implies that the internal energy of a gas is a function of temperature only (i.e. independent of its volume when the temperature is held constant).

The first law can be used to explain why rising air tends to cool whereas sinking air tends to warm. Suppose that we have a rising air parcel that is insulated from its environment such that no heat is added or taken away from the parcel. This is known as an adiabatic process. As the air parcel rises, it enters into regions of lower pressure. This causes the air parcel to expand since the lower pressure outside allows the air molecules to push out on the parcel walls. This means that air parcels is doing work on the environment at the expense of its own internal energy. Therefore, the parcel will cool as it rises. Conversely, a sinking parcel compresses since it is

Figure 3.8 (Left) Schematic of a rising air parcel. (Right) Schematic of a sinking air parcel.
moving into a region of higher pressure. Due to the parcel compression, the air molecules (which compose the air parcel) gains internal energy and thus, the parcel will warm.

3.3.4: Potential Temperature

Many atmospheric processes are close to adiabatic so it is useful to have a physical parameter that can be used as a tracer under adiabatic conditions. This variable is known as the potential temperature and it is given by

$$\theta = T \left( \frac{P_0}{P} \right)^{R_d/c_p}$$

where $T$ is the current temperature (in Kelvin), $R_d$ is the dry air gas constant, and $c_p$ is the specific heat at constant pressure (which is defined as the amount of heat required to raise the temperature of a 1 kg system by 1 degree Kelvin). The specific heat varies depending upon the system in question, but for air, the specific heat at constant pressure is experimentally verified to be $1004 \text{ J kg}^{-1} \text{ K}^{-1}$. The potential temperature of a parcel of fluid at pressure $P$ is the temperature that the parcel would acquire if adiabatically brought to a standard reference pressure $P_0$, usually 1000 millibars.

What is the use of potential temperature? Potential temperature can be used to compare the temperature of air parcels that are at different levels in the troposphere. As we have previously seen, temperature tends to decrease with height. This fact makes it more difficult to note which regions in the troposphere are experiencing warm air advection and cold air advection. Therefore, bringing air parcels adiabatically to a standard level (1000 millibars) allows comparisons to be made between air parcels at different elevations, similar to the idea of reduced sea-level pressure measurements. If the potential temperature of an air parcel at one pressure level is colder than air parcels at other pressure levels, a forecaster can infer cold air advection or a cold pocket exists at the pressure level with the lowest potential temperature.
Finding the potential temperature at a constant pressure level over an area produces one type of θ-chart, in which higher (lower) θ represents warmer (cooler) air. For example, θ can be found at 700 mb, as shown in Figure 3.9. Each location at 700 mb drops a parcel from the 700 to 1000 mb level and the temperature is read off at 1000 mb and thus, this is the 700 mb θ (θ is always given in degrees Kelvin).

A vertical cross section of θ can be produced by finding the areal distribution of θ at many pressure levels, then connecting the points of equal θ. At this point, sloping constant θ surfaces can be plotted. Air parcels tend to travel along constant Theta surfaces. This makes sense because constant θ surfaces represent “constant density” surfaces. The path of least resistance on an air parcel that is advecting is for it to remain at the same density as its environment. The term that describes this process is isentropic lifting / descent. Isentropic lifting (descent) occurs whenever warm air advection (cold air advection) or flow of one air mass over another occurs. Less dense air will tend to glide up and over more dense air (thus low level warm air advection leads to rising air) when less dense air advects toward more dense air.

Figure 3.9 700-mb θ map of the continental US at 22 December 2015 at 1200 Z.
Figure 3.10 290 K isentropic surface map of the continental US at 22 December 2015 at 1200 Z

You will hear isentropic (i.e. constant $\theta$) lifting referenced to often in forecast discussions. The trajectories that wind vectors take over isentropic surfaces determine how much lifting or sinking will take place due to advection. NWS forecasters are experts on these processes and use them as a major part of their forecasting process. The different ways of graphing $\theta$ can be quite complex, but the key points to remember are that: (1) air parcels in a convectively stable environment tend to advect along constant $\theta$ surfaces and (2) low level warm air advection produces isentropic lifting and uplift while cold air advection produces isentropic downglide and sinking.

While potential temperature can be used to compare temperatures at different elevations and the trajectory air parcels will take (rising or sinking), equivalent potential temperature can be used to compare BOTH moisture content and temperature of the air. The equivalent potential temperature $\theta_e$ is found by lowering an air parcel to the 1000 mb level AND releasing the latent heat in the parcel. The lifting of a parcel from its original pressure level to the upper levels of the troposphere will release the latent heat of condensation and freezing in that parcel. The more moisture the parcel contains the more latent heat that can be released. $\theta_e$ is used operationally to map out which regions have the most unstable and thus positively buoyant air. The $\theta_e$ of an air parcel increases with increasing temperature and increasing moisture content. Therefore, in a region with adequate instability, areas of relatively high $\theta_e$ (called $\theta_e$ ridges) are often the burst
points for thermodynamically induced thunderstorms and MCS’s. $\theta_e$ ridges can often be found in those areas experiencing the greatest warm air advection and moisture advection.

![Figure 3.11 700-mb $\theta_e$ map of the continental US at 22 December 2015 at 1200 Z](image)

**3.4: Kinematics of the Wind Field**

As can be readily discerned from inspection of any satellite animation or clouds or water vapor, the wind field varies zonally and meridionally. Even though it appears that there are an innumerable number of wind field patterns within the atmosphere at any given point in time, it turns out that the horizontal wind field can be decomposed into four types of flow patterns: *translation, divergence, vorticity, and deformation*. We will examine each type of flow pattern below.

**3.4.1: Derivation of Flow Patterns**

The physical properties of the horizontal wind field can be described by considering the Taylor-series expansion of the wind field about the point $(x_0, y_0)$. We will consider what happens to air parcels only near the origin so that higher-order nonlinear terms can be neglected. Therefore, we have

$$u(x, y) = u_0 + \left( \frac{\partial u}{\partial x} \right)_0 x + \left( \frac{\partial u}{\partial y} \right)_0 y + O(2),$$
Through algebraic manipulations, we have

\[ \nu(x, y) = \nu_0 + \left( \frac{\partial \nu}{\partial x} \right)_0 x + \left( \frac{\partial \nu}{\partial y} \right)_0 y + O(2), \]

Through algebraic manipulations, we have

\[ u(x, y) = u_0 + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial x} \right) - \left( \frac{\partial u}{\partial y} \right) + \left( \frac{\partial v}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right) \right] x + \frac{1}{2} \left[ \left( \frac{\partial u}{\partial y} \right) - \left( \frac{\partial v}{\partial x} \right) + \left( \frac{\partial u}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right) \right] y + O(2), \]

\[ \nu(x, y) = \nu_0 + \frac{1}{2} \left[ \left( \frac{\partial v}{\partial x} \right) - \left( \frac{\partial u}{\partial y} \right) + \left( \frac{\partial u}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right) \right] x + \frac{1}{2} \left[ \left( \frac{\partial v}{\partial y} \right) - \left( \frac{\partial u}{\partial x} \right) + \left( \frac{\partial u}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right) \right] y + O(2), \]

Defining

\[ D = \left( \frac{\partial u}{\partial x} \right) + \left( \frac{\partial v}{\partial y} \right), \quad \zeta = \left( \frac{\partial v}{\partial x} \right) - \left( \frac{\partial u}{\partial y} \right), \quad F_1 = \left( \frac{\partial u}{\partial x} \right) - \left( \frac{\partial v}{\partial y} \right), \quad F_2 = \left( \frac{\partial u}{\partial y} \right) + \left( \frac{\partial v}{\partial x} \right) \]

We have

\[ u(x, y) = u_0 + \frac{1}{2} (D + F_1)x - \frac{1}{2} (\zeta - F_2)y, \]

\[ \nu(x, y) = \nu_0 + \frac{1}{2} (\zeta + F_2)x - \frac{1}{2} (D - F_1)y \]

This analysis indicates that the horizontal wind field can be decomposed into four types of flow patterns: translation \((u_0, \nu_0)\), divergence \(D\), relative vertical vorticity \(\zeta\), and deformation \((F_1, F_2)\).

### 3.4.2: Translation

A wind field of pure uniform translation is given by Figure 3.12. For a field of pure translation, air parcels near the origin are translated downstream at the same rate of speed regardless of their initial location.
Figure 3.12: The effects of uniform translation on a fluid element

3.4.3: Divergence

Figure 3.13 (Left) A field of positive divergence. (Right) A field of negative divergence (i.e. convergence)
Figure 3.13 illustrates the wind field of pure divergence. The divergence provides us with a measure of how much the vector wind field is “spreading out” at each point. For a field of pure divergence, the area of air parcels is expanded as the fluid moves in all directions away from the origin. If the direction of the vector field is reversed, we obtain a wind field of pure convergence as the fluid moves in all directions towards the center. For a field of pure convergence, the area of air parcels is contracted independent of direction.

![Convergence and Divergence Diagram](image)

Figure 3.14 Schematic of convergence, divergence, confluence, and diffluence

Divergence in the wind field can be split into two components: **speed divergence** and **directional divergence**. Speed divergence is due to a change in wind speed along the direction of flow. Speed divergence is associated with a downstream increase of wind speed. Directional divergence (also called **diffluence**) is the directional spreading of the wind. The corresponding term for directional convergence is **confluence**. Diffluence (confluence) occurs when the flow spreads out (contracts) downstream (as shown in Figure 3.14). Thus, divergence is associated with a downstream increase in wind speed (i.e. stretching of air parcels) alone or with diffluence (i.e. spreading of air parcels) alone. A downstream decrease in wind speed, shrinking, coupled with diffluence may be divergent if the magnitude of the latter outweighs the former. Similarly, a downstream increase in wind speed, stretching, coupled with confluence, may be convergent if the latter outweighs the former. Observations suggest that synoptic-scale values of divergence are very small and consequently stretching is usually accompanied by confluence and shrinking is usually by diffluence. Therefore, it's not correct to associate diffluence with divergence and confluence with convergence as a general rule.
3.4.4: Vorticity

Figure 3.15 illustrates a wind field of pure vorticity. As is shown, vorticity is associated with the local rotation of the air flow. In the Northern Hemisphere, a counterclockwise rotation of the air is associated with positive (or cyclonic) vorticity, whereas a clockwise rotation of the air is associated with negative (or anticyclonic) vorticity. Therefore, **cyclonic vorticity** is characterized by a component of rotation about the local vertical that is in the same direction as the Earth's axis of rotation, whereas **anticyclonic vorticity** is characterized by a component of rotation about the local vertical that is in the opposite direction as the Earth's axis of rotation. Since the rate of rotation is uniform (independent of direction), the area and shape of the air parcel are preserved. Vorticity is also designed as **cyclonic** and **anticyclonic**.
Similar to divergence, vorticity in the wind field can be split into two components: curvature vorticity and shear vorticity (as shown in Figure 3.16). **Curvature vorticity** is associated with a change in wind direction over some horizontal distance. This change will result in either a counter-clockwise or clockwise curvature. **Shear vorticity** is associated with a change in speed over some horizontal distance. Unlike the individual contributions from each component for divergence, the individual contributions from each term for vorticity do not act in the opposite sense for synoptic-scale motions. Positive vorticity can be produced by

- Wind speed increasing when moving away from the center point of a trough or low (positive shear vorticity).
- A counterclockwise curvature in the wind flow, which occurs in troughs or lows (positive curvature vorticity).

Negative vorticity can be produced by

- Wind speed decreasing when moving away from the center point of a trough (negative shear vorticity).
- A clockwise curvature in the wind flow, which occurs in ridges or highs (negative curvature vorticity).
Cyclonic shear poleward of a trough in the westerlies enhances the cyclonic vorticity associated with curvature, while anticyclonic shear equatorward of the trough counteracts the cyclonic curvature vorticity. Similarly, cyclonic shear poleward of a ridge in the westerlies counteracts the anticyclonic vorticity associated with curvature, while anticyclonic shear equatorward of the ridges enhances the anticyclonic curvature vorticity.

### 3.4.5: Deformation

Figure 3.17 (Left) A field of pure, positive stretching deformation. (Right) A field of pure, positive shearing deformation

Figure 3.17 demonstrates a wind field of pure stretching deformation. This flow field pattern is stretched along the $x$-axis and compressed along the $y$-axis. In fact, these two axes have special names: the flow is stretched along the **axis of dilatation** while it is compressed along the **axis of contraction**.
It is important to distinguish between deformation and convergence as they are commonly confused. If we consider the area of an air parcel bounded by curve $C$ embedded within a field of pure convergence as shown in Figure 3.18, we see immediately that the area will become progressively smaller under the influence of the convergent flow. If the same fluid parcel were placed in a field of pure stretching deformation, however, the shape of the originally square air parcel would be deformed into a rectangle while preserving its area.

Figure 3.17 also demonstrates a wind field of pure shearing deformation. For a field of pure shearing deformation, the flow is similar to the stretching deformation rotated counterclockwise by 45°. We are usually most interested in the total deformation field given by

$$F = \sqrt{F_1^2 + F_2^2}$$

Unlike divergence, deformation on the synoptic-scale is not necessarily very small. Whereas the effects of stretching and confluence/diffluence counteract each other as far as divergence is concerned, the effects of stretching and confluence/diffluence usually act in concert. As we will see in the next chapter, deformation tends to be extremely important for intensifying surface fronts.
Chapter 4: Synoptic Fronts

Inspection of satellite imagery frequently reveals long, narrow cloud bands that often correspond to frontal zones. Why do fronts matter? Whether the systematic lowering and thickening of stratiform clouds accompanying the approach of a warm front, or the formation of convective storms along a cold front, it is clear that fronts can strongly influence weather conditions. For this reason, forecasts must account for their movement, type, intensity, and influence on cloud and precipitation. Depending on the static stability of air in the vicinity of a frontal system, fronts can trigger severe, organized convective storms. The presence of enhanced vertical wind shear in frontal zones contributes to the organization and severity of convective storms that form in their vicinity. In some situations, convective storms can develop in succession along frontal boundaries, resulting in flash flooding as sequential cells move over the same areas repeatedly. Both the horizontal wind shift and vertical wind shear accompanying frontal zones have important implications for aviation forecasting, and the timing of fronts can be critical to determining the amount and type of precipitation in a given location.

To understand and predict frontal weather, we must first understand the mechanisms that lead to frontal formation and structure. We begin the chapter with a discussion of introductory thermodynamics.

4.1: Air Masses

The Glossary of Meteorology defines a front as the “interface or transition zone between two air masses of different density”. This requires use of a density variable, such as potential temperature or density, to identify a front. Since fronts are defined as the boundary between two regions of contrasting air density, it's helpful to examine air masses. Air masses are traditionally classified according to the scheme developed by Tor Bergeron in 1928. This sorts air masses by moisture (continental, c; or maritime, m) and temperature (arctic, A; polar P; tropical T; or equatorial E). The air mass may be further classified according to whether it is significantly warmer or colder than the surface it is passing over. An air mass that is warmer than Earth's surface gains the suffix “w”. Since it is being cooled from below, it is prone to forming a stable inversion near the surface, favoring haze and leading to stratus and fog if it has high moisture content. Likewise an air mass that is cooler than the Earth's surface has the suffix “k”, which means that it is being heated from below. A “k” air mass is destabilizing, and as a result it is prone to wind gusts, low-level wind shear, and cumuliform clouds.
Once an air mass is identified, its abbreviation may be written near the center of the mass. The abbreviation is comprised of the moisture letter, the temperature letter, and the relative temperature letter if applicable. This annotation communicates basic information about the air mass to other forecasters and end user. Again it must be remembered that these classifications are idealized air mass archetypes, and air mass may originate from other regions and have ambiguous characteristics. The primary archetypes are as follows: continental polar (cP), continental tropical (cT), maritime polar (mP), maritime tropical (mT), and arctic (A), as shown in Figure 4.1.

Continental polar (cP) air forms due to radiational cooling of air over cold or frozen terrain. Its most common source in North America is far northern Canada and the Arctic basin. In the winter, source regions can extend south into the United States if fresh snow cover blankets a large region. When cP air reaches the warm waters of the Gulf of Mexico and Atlantic, it becomes cPk and forms extensive streets of stratocumulus and cumulus clouds.

Continental tropical (cT) air is associated with strong heating of dry terrain by the sun. Its most common source region is Arizona, New Mexico, and northern Mexico, especially during the spring and summer. The most common characteristics are fair skies, warm temperatures, and low dew points. The dryline, a surface feature found in the Great Plains, is a boundary separating this type of air from maritime tropical air, and where strong westerlies carry the light buoyant air eastward above the tropical air mass it is known as an elevated mixed layer (EML) and forms a
capping inversion at the interface beneath. In the 1930s and 1940s, this EML was occasionally classified as an air mass type called Superior. The northern incursion of maritime air in late summer causes the onset of monsoon rains in the Desert Southwest.

Maritime polar (mP) air forms when air masses stagnate over cool ocean surfaces. It tends to originate over Asia as cP air and tends to be unstable. The temperature of an mP air mass is similar to that of the cool ocean waters it rests over, usually from 30 to 60°F. Nearly all weather systems that come ashore on the west coast of the United States usher in maritime polar air. As shown in Figure 4.2, mP air is modified by the time it reaches the interior of the US, although it is milder than cP air.

Maritime tropical (mT) air is found across all tropical oceans, and due to the high evaporation of these warm waters the air mass goes large values of dew point -- 60 to 80°F. Maritime tropical air contains abundant and rich moisture, but since the ocean absorbs most of the incoming solar energy, convective instability tends to be weak. When this air mass is advected onshore as a mTk air mass, it tends to gain considerably more heat and it destabilizes further, producing clouds and thunderstorms. Since tropical oceans cover a substantial percentage of the globe, maritime tropical air is the single most common air mass. Furthermore, the strong solar heating of land masses in the Tropics allows extensive infiltration of this air inland. Even when the air mass moves inland, evapotranspiration from damp tropical terrain and vegetation helps offset the moisture lost from storm clouds.
4.2: General Characteristics of Synoptic Fronts

Figure 4.3 Schematic of cold and warm fronts

Since enhanced horizontal gradients of temperature associated with frontal zones are also zones of strong vertical wind shear by thermal wind balance, then strong and deep frontal zones would necessarily accompany strong upper-level winds. We will see in a future chapter that these frontal zones are accompanied with the midlatitude jet stream. Moreover, one would expect strong vertical circulations to be associated with frontal zones, as air parcels moving into or out of the frontal zone would experience a marked change in the magnitude of the horizontal temperature gradient. On a basic level, this is one reason why fronts are often associated with bands of cloud and precipitation.

Noting that frontal zones are long and narrow, we can define spatial scales characteristic of frontal zones as synoptic scale (1000 km) in the along-front direction, and one order of magnitude less than this in the cross-front direction (100 km). Therefore, the cross-front horizontal scale is therefore decidedly mesoscale. Owing to their along-front dimension, and also because these air mass boundaries were originally analyzed on early synoptic maps, we shall refer to these boundaries as synoptic fronts in order to distinguish them from the other air mass boundaries that are fully meridional (such as drylines, gust fronts, etc.). Observations demonstrate that fronts display the following characteristics:
• Enhanced horizontal contrasts of temperature and/or moisture
• A relative minimum of pressure (trough) and maximum of cyclonic vorticity along the front;
• Strong vertical wind shear, and horizontal wind shift consistent with a pressure trough and cyclonic vorticity;
• Large static stability within the frontal zone;
• Ascending air, clouds, and precipitation near the front (depending on moisture availability and other factors); and
• Greatest intensity near the surface, weakening with height

A synoptic front whose intensity is strongest near the ground is called a **surface front**. Surface fronts usually are usually formed downstream from upper-level troughs and upstream from upper-level ridges. Intense frontal zones are not only found at the surface; observational and theoretical studies have documented the formation of strong fronts near the tropopause as well. The strong vertical wind shear accompanying these systems can result in clear-air turbulence, which is of interest to the aviation industry, among others. In this chapter, we will focus our attention on surface fronts.

### 4.3: Frontogenesis

**Frontogenesis** refers to an increase in the magnitude of the horizontal density gradient, whereas **frontolysis** refers to a decrease in the magnitude of the horizontal density gradient. A traditional measure of frontogenesis was introduced by Norwegian meteorologist, Sverre Pettersen, to explore the kinematic processes leading to change in the strength of the gradient of a scalar field following a moving air parcel. If we choose potential temperature as our scalar field, the **frontogenesis function** \( F \) is defined as the time rate of change of the magnitude of the horizontal potential temperature gradient following the flow. More generally, for any orientation of coordinate axes and for cases in which an along-front potential temperature gradient is present, \( F = d/dt |\nabla_h \theta| \). The advantage of defining \( F \) with potential temperature, rather than temperature, is that in regions of complex terrain, the potential temperature can help to isolate synoptic features (including fronts) by correcting the temperature to a common pressure level, thereby reducing the portion of the temperature gradient due to elevation. Also, potential temperature is a conservative variable for adiabatic flow.
Here, the different physical mechanisms that can lead to changes in the potential temperature gradient following the flow will be examined on a conceptual level. Pettersen demonstrated that the frontogenesis function can be written as

\[ F_{\text{equiv}} = \frac{d}{dt} |\nabla_h \theta| = F_{\text{shear}} + F_{\text{stretch}} + F_{\text{tilt}} + F_{\text{heat}} \]

Figure 4.4 Schematic of the effect of \( F_{\text{shear}} \). Top panel depicts shearing frontolysis along the warm front, whereas bottom panel depicts frontogenesis along the cold front.

\( F_{\text{shear}} \) represents the effects of horizontal shear on the horizontal temperature gradient. Shearing frontogenesis describes the change in frontal strength due to differential potential temperature advection by the front-parallel wind component. For shearing frontogenesis, a strengthened temperature gradient results from cold advection in the cold air and warm advection in the warm, both of which act to increase the frontal contrast. Shearing can also have a frontolytic effect when warm advection occurs within cold air or cold advection occurs within warm air, both of which act to decrease the frontal contrast.
Figure 4.5 Schematic of the effect of \( F_{\text{stretch}} \), depicting frontogenesis.

\( F_{\text{stretch}} \) represents the kinematic effect of confluence/diffluence on the horizontal temperature gradient. Confluence acts to increase the horizontal potential temperature gradient, while diffluence acts to decrease the horizontal potential temperature gradient. Confluence and diffluence contribute to both convergence (divergence) and horizontal (nondivergent) deformation. Because of the smaller cross-front distance scale, the confluence term tends to be larger than the shearing term.

Figure 4.6 Schematic of the effect of \( F_{\text{tilt}} \), depicting frontogenesis.

\( F_{\text{tilt}} \) represents kinematically the tilting of the vertical potential temperature gradient onto the horizontal. In a statically stable atmosphere, rising motion and its associated adiabatic cooling on the cold side, and sinking motion and its associated adiabatic warming on the warm side, increase the temperature gradient. Near the Earth's surface, the vertical motion is small, and the effect of tilting is often relatively weak.
Figure 4.7 Schematic of the effect of $F_{\text{heat}}$, depicting frontogenesis.

$F_{\text{heat}}$ represents a horizontal variation in diabatic heating. For example, heating of the warm side of a front by the sun during the day, if it is clear there, without heating on the cold, cloudy side, if it is cloudy there, is a frontogenetical process. At night the effect of long-wave cooling is dominant, so that cooling on the clear, warm side, with restricted cooling on the cold, cloudy side, represents a frontolytical process. Differential heating along the boundary between snow cover and bare ground can act frontogenetically. If a surface cold front is very shallow, and relatively warm air is present above the shallow cold air mass, then heating by the sun may result in the total destruction of the cold air mass, and hence in the destruction of the front. In the High Plains area of the United States, a shallow cold front may be turned into a dryline by this mechanism.

**4.3.1: Derivation of Frontogenesis Function**

To examine frontogenesis, we need to examine the evolution of $\theta$, which means that we need to derive an expression for the rate of change of $\theta$. The first law of thermodynamics is usually derived by considering a system in thermodynamic equilibrium, that is, a system that is initially at rest and after exchanging heat with its surroundings and doing work on the surroundings is again at rest. For such a system the first law states that the change in internal energy of the system is equal to the difference between the heat added to the system and the work done by the system. A Lagrangian control volume consisting of a specified mass of fluid may be regarded as a thermodynamic system. However, unless the fluid is at rest, it will not be in thermodynamic equilibrium. Nevertheless, the first law of thermodynamics still applies. To show that this is the case, we note that the total thermodynamic energy of the control volume is considered to consist of the sum of the internal energy (due to the kinetic energy of the individual molecules) and the kinetic energy due to the macroscopic motion of the fluid. The rate of change of this total thermodynamic energy is equal to the rate of diabatic heating plus the rate at which work is done on the fluid parcel by external forces.
If we let \( e \) designate the internal energy per unit mass, then the total thermodynamic energy contained in a Lagrangian fluid element of density \( \rho \) and volume \( \delta V \) is
\[
\rho \left[ e + \frac{1}{2} U^2 \right] \delta V.
\]
In the absence of viscous forces, the external forces that act on a fluid element are the pressure force, the gravitational force, and the Coriolis force. Our job is to determine the rate at which work is done on the fluid element by each of these forces.

First, we note that the Coriolis force is perpendicular to the velocity vector, it can do no work. Second, we note the rate at which the gravitational force does work on the mass element is
\[
\rho g w \delta V,
\]
where \( w \) is the vertical velocity. The rate at which work is done on the fluid element by the \( x \) component of the pressure force is illustrated in Figure 4.8. Recall from introductory physics that pressure is force per unit area and that the rate at which a force does work (i.e. the power) is given by \( \vec{F} \cdot \vec{U} \). We see that the rate at which the surrounding fluid does work on the element due to the pressure force on the two boundary surfaces in the \( y,z \) plane is given by
\[
(Pu)_A \delta y \delta z - (Pu)_B \delta y \delta z.
\]
Note that the negative sign is needed before the second term because the work done on the fluid element is positive if \( u \) is negative across face B. Now by expanding in a power series we can write
\[
(Pu)_B = (Pu)_A + \left[ \frac{\partial}{\partial x} (Pu) \right]_A \delta x + \cdots
\]
Thus, the net rate at which the pressure force does work due to the \( x \) component of motion is
\[(Pu)_A - (Pu)_B \delta y \delta z = - \left[ \frac{\partial}{\partial x} (Pu) \right]_A \delta V\]

Generalizing to three dimensions, we have that the total rate at which work is done by the pressure force is

\[-\nabla \cdot (P\bar{U}) \delta V\]

Applying the principle of energy conservation to our Lagrangian control volume (neglecting effects of molecular viscosity), we thus obtain

\[
\frac{D}{Dt} \left[ \rho \left( e + \frac{1}{2} U^2 \right) \right] = -\nabla \cdot (P\bar{U}) \delta V + \rho gw \delta V + \rho \dot{Q} \delta V
\]

Here, \( \dot{Q} \) is the rate of heating per unit mass due to radiation, conduction, and latent heat release. Using the chain rule, we can rewrite the above expression as

\[
\rho \delta V \frac{D}{Dt} \left( e + \frac{1}{2} U^2 \right) + \left( e + \frac{1}{2} U^2 \right) \frac{D(\rho \delta V)}{Dt} = -\bar{U} \cdot \nabla P \delta V - P\bar{U} \cdot \bar{U} \delta V - \rho gw \delta V + \rho \dot{Q} \delta V
\]

Now based on the mass continuity equation, the second term on the left hand side of the expression vanishes so that we have

\[
\rho \frac{De}{Dt} + \rho \frac{D}{Dt} \left( \frac{1}{2} U^2 \right) = -\bar{U} \cdot \nabla P - P\bar{U} \cdot \bar{U} - \rho gw + \rho \dot{Q}
\]

We can simplify by this expression by using the vector momentum equation. If we take the dot product of \( \bar{U} \) with the momentum equation, we obtain

\[
\bar{U} \cdot \frac{D\bar{U}}{Dt} = -\frac{1}{\rho} \bar{U} \cdot \nabla P + g \cdot \bar{U} \Rightarrow \rho \frac{D}{Dt} \left( \frac{1}{2} U^2 \right) = -\bar{U} \cdot \nabla P + \rho gw
\]

Using this in our expression for energy conservation gives

\[
\rho \frac{De}{Dt} = -\rho \nabla \cdot \bar{U} + \rho \dot{Q}
\]

This is known as the thermal energy equation. This can be written in a more familiar form by using the continuity equation. From the continuity equation, we note that
\[ \frac{1}{\rho} \nabla \cdot \vec{U} = -\frac{1}{\rho^2} \frac{D\rho}{Dt} = \frac{D\alpha}{Dt} \]

where \( \alpha = \rho^{-1} \) is known as the specific volume. We also note that for dry air, the internal energy per unit mass is given by \( e = c_v T \), where \( c_v \) is the specific heat at constant volume. This gives us

\[ c_v \frac{DT}{Dt} + P \frac{D\alpha}{Dt} = \dot{Q} \]

This is known as the **thermodynamic energy equation**, which is the first law of thermodynamics applied to a moving fluid.

We can write the thermodynamic energy equation in terms of potential temperature \( \theta \). First, taking the total derivative of the ideal gas law (in terms of specific volume) gives

\[ \frac{D}{Dt} (P\alpha) = R_d \frac{DT}{Dt} \Rightarrow P \frac{D\alpha}{Dt} = R_d \frac{DT}{Dt} - \alpha \frac{DP}{Dt} \]

Substituting this into the thermodynamic energy equation and noting that \( c_p = c_v + R_d \), we have

\[ c_p \frac{DT}{dt} - \alpha \frac{DP}{dt} = \dot{Q} \]

Dividing both sides by \( T \) and using the ideal gas law, we obtain

\[ c_p \frac{D \ln T}{Dt} - R_d \frac{D \ln P}{Dt} = \frac{\dot{Q}}{T} \]

Now, by the definition of potential temperature

\[ \theta = T \left( \frac{P_0}{P} \right)^{R_d/c_p} \Rightarrow \ln \theta = \ln T + R_d \left( \ln P_0 - \ln P \right) \Rightarrow c_p \frac{D \ln \theta}{Dt} = c_p \frac{D \ln T}{Dt} - R_d \frac{D \ln P}{Dt} \]

Therefore, we have

\[ c_p \frac{D \ln \theta}{Dt} = \frac{\dot{Q}}{T} \Rightarrow \frac{D \theta}{Dt} = \frac{\dot{Q} \theta}{c_p T} \]
By differentiating with respect to \(y\) (i.e. the along front direction), we obtain the equation for the time rate of change of the strength of baroclinicity along a front

\[
F = \frac{d}{dt} \left( -\frac{\partial \theta}{\partial y} \right) = \frac{\partial u}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial v}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial w}{\partial y} \frac{\partial \theta}{\partial z} - \frac{\partial}{\partial y} \left( \frac{\dot{Q}_\theta}{c_p T} \right)
\]

where \(F\) is known as the scalar frontogenesis function.

The first term is known as the shearing frontogenesis term. Shear frontogenesis describes the change in front strength due to differential temperature advection by the front-parallel wind component. Along the cold front given in the bottom panel of Figure 4.4, both \(\partial u/\partial y\) and \(\partial \theta/\partial x\) are negative, giving a positive contribution to \(F\). This means cold (warm)-air advection in the cold (warm) air. Along the warm front in the top panel of Figure 4.4, \(\partial \theta/\partial x\) is positive, but \(\partial u/\partial y\) is negative, giving a negative contribution to \(F\). This means along the warm front, shearing acts in a frontolytical sense.

The second term is the confluence or stretching term. Confluence frontogenesis describes the change in front strength due to stretching. Along the front in Figure 4.5, both \(\partial \theta/\partial y\) and \(\partial v/\partial y\) are negative, giving a positive contribution to \(F\). This means along the front, confluence acts in a frontogenetical sense. The third term is the tilting term and it usually acts to strengthen fronts above the Earth’s surface, as in Figure 4.6. In that figure, \(\partial \theta/\partial z\) is negative (temperature decreases above the surface), and \(\partial w/\partial y\) is also negative (rising motion in the cold air, sinking in the warm air), leading to frontogenesis.

The fourth term is the differential diabatic heating term, which takes into account all diabatic processes together (such as differential solar radiation, differential surface heating due to soil characteristics, differential heat surface flux, etc.). In Figure 4.7, we see the effects of differential solar radiation. Based on the figure, the diabatic heating rate in the warm air exceeds the diabatic heating rate in the cold air. This makes \(\partial/\partial y(\dot{Q}_\theta/c_p T)\) positive, causing frontogenesis.

### 4.3.2: Frontogenesis and Deformation

If the effects of tilting and differential diabatic heating can be neglected, then the frontogenesis function can be written as

\[
F = \frac{d}{dt} |\nabla_h \theta| = \frac{\nabla_h \theta}{2} (D \cos 2\beta - \delta)
\]
where \( \beta \) is the angle between the isentropes (i.e. lines of constant \( \theta \)) and the axis of dilatation, \( \delta \) is the horizontal divergence, and \( D \) is the total deformation. We observe from the above equation that frontogenesis occurs whenever a nonzero horizontal potential temperature gradient coincides with convergence \( \delta < 0 \), and whenever the total deformation field acts upon isentropes that are oriented within \( 45^\circ \) of the axis of dilatation, as shown in Figure 4.8. For angles between isentropes and the axis of dilatation that are between \( 45^\circ \) and \( 90^\circ \), frontolysis occurs. Divergence \( (\delta > 0) \) also contributes to frontolysis. Vorticity does not contribute directly to frontogenesis or frontolysis, but can affect frontogenesis or frontolysis by rotating the isentropes, thereby changing the angle between the isentropes and the axis of dilatation, which affects \( F \). This approximation essentially states that the geometry of the horizontal flow has a strong influence on frontogenesis in most situation and thus the two main processes (parameters) that make significant separate contributions to frontogenesis is divergence and deformation.

![Figure 4.9 Schematic illustration of the relationship between isentropes (blue contours) and an idealized wind field dominated by deformation (red streamlines) in a situation of (a) frontogenesis and (b) frontolysis. The axis of dilatation (black dotted line) and angle \( \beta \) are also indicated.](image)

Now that we have reviewed the general properties of fronts and have examined the kinematic processes responsible for their evolution, it remains to illustrate examples of different types of frontal structure and associated weather. Our goal is to extend beyond a basic discussion of the main frontal types — to focus on some of the key structural differences leading to contrasting weather in their vicinity. The basic frontal types — cold, warm, stationary, and occluded — are loosely defined based on the sense of local movement and temperature tendency in the frontal zone. Fronts that are moving slowly can be categorized as stationary fronts, and other fronts may require examination of sequential analyses to discern their movement. The mechanism of
movement of fronts varies with type, but the frontal kinematics outlined in the preceding section is generally relevant to all types of fronts.
4.4: Cold Fronts

![Diagram of New England back-door cold front](image)

Figure 4.10 Example of a New England back-door cold front 0000 UTC, April 29, 1990.

When a cold air mass advances equatorward, equatorward and eastward, or eastward relative to the warm air mass, the front is called a **cold front**. When the cold air advances westward or equatorward and westward, the front is called a **back-door cold front** because synoptic-scale weather systems in the midlatitudes usually have an eastward component of motion. The former usually trail off in an equatorward direction from a surface cyclone, and are followed by a cold, surface anticyclone. Synoptic-scale vertical motion is usually downward, and an upper-level ridge is found upstream with respect to the flow aloft. Back-door cold fronts are common in the northeastern part of the United States during the spring and summer, east of the Rocky Mountains any time of year, and sometimes in the Great Plains during the period from fall through spring. The spring and summer New England back-door cold front is influenced strongly by the relatively cold ocean upstream.
The surface frontal zone (i.e., band of surface temperature gradient much greater in magnitude than the synoptic-scale horizontal temperature gradient) is usually found on the cold side of the wind shift associated with a surface pressure trough. The wind veers (backs) along the wind-shift line equatorward (poleward) of the cyclone. It is common practice to identify the front symbol (line along which there are toothlike triangles pointing from the cold side toward the warm side) with the wind-shift line along which there is a maximum in cyclonic vorticity. Just as or shortly after the wind has shifted, the temperature drops and the pressure rises. One is in the frontal zone from the time the wind shifts direction and the temperature begins to drop until the time the temperature no longer falls rapidly.
Figure 4.12 Examples of synoptic fronts as seen in approximately front-normal vertical cross-section of potential temperature (blue contours). (a) Cold front analyzed by Sanders (1955) using rawinsonde observations. (b) Strong cold front analyzed by Shapiro et al. (1985) based on observations obtained from an instrumental tower (observations were time-to-space converted). (c) Warm front analyzed by Bluestein (1993) using rawinsonde observations.

The surface front is most intense at the ground, and weakens with height. It can intensify dramatically on time scales as short as 12 hours. The frontal zone is marked by strong static stability; it is associated with strong vertical wind shear, which is associated to some extent with the surface temperature gradient, in accord with the thermal wind relation. The frontal zone is steepest near the ground. A narrow jet of rising air is often located just above the leading edge of the surface front. A narrow cloud line ("rope cloud") or band of precipitation may be located along the rising jet if there is sufficient moisture. Below the frontal zone in the cold air at low levels, the lapse rate (i.e. the change in temperature with height) may be nearly adiabatic, and hence there is strong turbulent mixing that creates gusty surface winds.

The movement of a cold front is not necessarily determined by the difference between the wind components normal to the front on either side of it. Movement is best correlated with the wind component normal to the front in the cold air. The movement of the surface cold front should be dependent upon the front-normal isallobaric gradient. The trough associated with a frontal zone moves from a region where surface pressures are rising to a region where surface pressures are falling. Because the zone of temperature gradient is located on the cold side of the wind-shift line, we expect to find strong cold advection there. Moreover, the cold advection is
associated with sinking motion and surface pressure rises. Thus, it's not surprising that frontal movement is associated with the front-normal wind speed in the cold air. The distribution of clouds on either side of a front depends upon the vertical motion, the front-relative flow, and the availability of moisture upstream from the front. The front-relative flow in eastward-moving fronts is likely to be rear-to-front, owing to strong westerlies aloft. The front-relative flow in equatorward-moving fronts is more likely to be front to rear, especially if there is a poleward component of flow aloft downstream from a trough. Frontal motion can also be influenced by diabatic effects such as latent cooling resulting from the evaporation of precipitation (latent cooling influences the surface pressure field and therefore also affects the isallobaric gradient).

### 4.4.1: Katafront and Anafront Structure

![Diagram of kata and ana cold fronts](image)

Figure 4.13 Comparing kata cold front and ana cold front

Broadly speaking, cold fronts can be divided into two types: **ana** and **kata** cold fronts. These types can be described both in terms of classical frontal theory and in terms of conveyor belts. The main feature which separates the different types of cold front is the orientation of the jet relative to the front in the middle and upper levels of the troposphere: In the case of an ana cold front, the jet axis and dry intrusion are parallel to the frontal cloud band, and form a well pronounced rear cloud edge. In the case of a kata cold front, the jet axis crosses the frontal cloud band.
For ana cold fronts, the cold air moves rapidly against warm air, creating convergence within the baroclinic zone between the two air masses. Convergence forces the warm, moist air to ascend along the frontal surface. The developing cloud band is inclined rearward with height. The main zone of cloudiness and precipitation is located behind the surface front. The frontal cloud band and precipitation are related to an ascending **warm conveyor belt**, which has a rearward component relative to the movement of the front, causing the frontal cloud band and precipitation to appear behind the surface front. Parallel to the warm conveyor belt there is a dry stream (dry intrusion). The sharp rear cloud edge of frontal cloudiness marks the transition between the two relative streams.
For kata cold fronts, the ascent of warm air is restricted by dry descending air originating from behind the front and, consequently, dissipating the higher clouds. The main zones of cloudiness and precipitation appear in front of the surface front. Unlike the ana cold front, the ascending warm conveyor belt is overrun by the dry intrusion. The dry air originates from upper levels of the troposphere and crosses the cold front from behind. The warm conveyor belt acquires a component which is inclined forwards relative to the movement of the cold front. Therefore, frontal clouds and precipitation tend to lie ahead of the surface front. The cloud tops in the area of the dry airstream are relatively low, whereas on the leading edge of this area the cloud tops are higher. This area indicates the so-called upper Cold Front. The air mass which is advected by the dry intrusion is colder than the air within the warm conveyor belt. The intrusion cools air above and, later, also ahead of the cold front.

4.5: Warm Fronts

When the warm air mass advances poleward, poleward and eastward, and sometimes eastward relative to the cold air mass, the surface front is called a warm front. The poleward or poleward and eastward movement of a warm front is usually associated with strong low-level warm advection east or poleward and east of a developing or intensifying surface cyclone. However, not all surface cyclones are associated with warm fronts, or with warm fronts with well-defined wind-shift lines.
The wind veers when a warm front passes. Low clouds and fog often clear away, as drier air not having a history of ascent arrives. In the Southern Plains, however, warm frontal passage may mark the return of stratocumulus overcast from the Gulf of Mexico. Relatively warm, Atlantic air sometimes circulates from the east around the north side of a cyclone near the Great Lakes and eventually advances westward, southwestward, southward, and even southeastward. This feature could be called a **back-door warm front**.

In this High Plains of the United States, the passage of a lee trough usually marks the shift to dry, potentially warm air from the west. This warming along the leading edge of the downslope air may be rather pronounced, especially when the air mass being displaced is cold, Arctic air that had arrived after the passage of a Norther. This type of warm front is sometimes referred to as a **chinook front**. Strong downslope wind storms are common in the foothills of the Rockies as the lee trough/chinook front passes east of the Continental Divide. The potential for damaging winds is a function of wind direction, vertical wind profile, and static stability upstream.

![Figure 4.16 Schematic of warm front using conveyor belt theory](image)

The idealized structure and physical background of a warm front can be explained with the conveyor belt theory. Frontal cloud band and precipitation are in general determined by the ascending warm conveyor belt, which has its greatest upward motion between 700 and 500 hPa. The warm conveyor belt starts behind the frontal surface in the lower levels of the troposphere, crosses the surface front and rises to the upper levels of the troposphere. There the warm conveyor belt turns to the right (anticyclonically) and stops rising, when the relative wind turns to a direction parallel to the front. If there is enough humidity in the atmosphere, the result of this ascending Warm Conveyor Belt is condensation and more and more higher cloudiness.

The cold conveyor belt in the lower layers, approaching the warm front perpendicularly in a descending motion, turns immediately in front of the surface Warm Front parallel to the surface front line. From there on the cold conveyor belt ascends parallel to the warm front below.
the warm conveyor belt. Due to the evaporation of the precipitation from the warm conveyor belt within the dry air of the cold conveyor belt, the latter quickly becomes moister and saturation may occur with the consequence of a possible merging of the cloud systems of warm and cold Conveyor belt to form a dense nimbostratus.

Generally speaking, warm fronts are weaker than cold fronts. This implies that warm fronts are usually less steeply sloped than cold fronts and tend to have smaller temperature contrasts than cold fronts. Moreover, the wind shift along warm fronts is often not as pronounced as along cold fronts, which is related to the fact that warm fronts also tend to be have weaker temperature gradients than cold fronts.

### 4.6: Stationary Fronts and Occlusions

When neither the cold air mass nor the warm air mass advances much relative to each other, the front that separates the two is referred to as stationary. Rising motion above the frontal zone is typically associated with warm advection. This brand of rising motion is often referred to as overrunning, since the warm air mass overruns the cool air mass. Precipitation associated with overrunning is usually stratiform and light owing to the high static stability in the frontal zone.
As cold fronts slow to a halt, often as a result of cyclogenesis to the west or equatorward and to the west, they become stationary. It is common for the stationary front to move back poleward as a warm front as the new cyclone deepens and moves poleward and eastward. When precipitation falls along and north of a stationary front, flooding is sometimes possible, since the region experiencing precipitation may do so for a long time.
Figure 4.19 Basic evolution of a cold occlusion and warm occlusion
According to polar-front theory, if a cold front overtakes a warm front equatorward of a cyclone the resulting wind and temperature field is referred to as an **occlusion**, since the warm air at the surface is blocked off from the ground. The surface boundary along which the cold front meets the warm front is called an **occluded front**. For an occlusion, the warm air mass is pinched in between the upstream and downstream cold air masses. Occluded fronts in which the advancing cold air is warmer than the retreating cold air are called **warm occlusions**. **Cold occlusions** are occluded fronts in which the advancing cold air is colder than the retreating cold air.

Occluded fronts that move across the Pacific Northwest and then downslope east of the Rockies are usually warm occlusions. A relatively mild, maritime air mass is warmed through adiabatic compression in the lee of the mountains and advances relative to a retreating, cold, continental air mass. These occluded fronts are called **trowals** in Canada. In the lee of the mountains, these fronts may behave more like warm fronts. Occluded fronts in the eastern United States are often cold occlusions, as relatively cold, continental air advances relative to retreating warm Atlantic air.

However, verification of the classic occlusion through analysis of observational data has been difficult! There are currently several other conceptual models for the formation of occluded fronts, which would go beyond the scope of these notes.

### 4.7: Coastal Fronts

A common feature along the southeastern U.S. coast, the Texas coast, and in coastal New England is a frontal zone that forms preferentially along the coast. The **coastal front** is a shallow (i.e. less than 1 km deep) mesoscale zone of strong horizontal temperature gradient (on the order of 10°C/10 km) at the surface that separates relatively warm, maritime air from cold, continental air. The coastal front has a time scale of 6 – 12 hours, which is somewhat less than the synoptic time scale of days. Coastal fronts are often found during the early and middle winter along the east coast of New England and the Carolina and Texas coasts, all of which are curved in a concave manner. During the early winter, there can be a substantial land-sea temperature contrast, because the ocean temperature is still relatively warm compared to the air temperatures over land. When cold, continental air flow offshore along a concave coastline, there is a relative maximum in diabatic heating.
Coastal fronts usually behave like stationary fronts or warm fronts. They often move toward the cold air mass, and are marked by a convergence zone, along which there is a shift in wind direction. The maximum amount of accumulated precipitation is found along a band on the cold side of the front. Sometimes the coastal front separates frozen precipitation (on the cold side) from rain (along the warm side), or heavy precipitation from light precipitation.

Coastal frontogenesis in New England occurs when there is an anticyclone associated with a cold air mass to the north or northeast of New England, and when a trough approaches from the west at low to middle levels. The cold air flows southward and southwestward, and my become trapped by the Appalachian mountains. This phenomenon is known also as cold air damming, since the relatively dense, cold air cannot be lifted easily to the west over the mountains: Less energy is required to deflect the airflow around the mountains. The quasistationary ridge that forms as a hydrostatic consequence of the trapped cold air mass is often referred to as the Baker ridge.
The combination of cold air over the land and relatively warm air over the ocean produces a land-to-ocean directed horizontal temperature gradient. Although differential friction between the land and ocean can account for a band of convergence when there is onshore flow, it is not generally considered to be significant. In addition, heating of the cold, continental air from below by the relatively warm ocean results in land breeze, in which an offshore wind component along the shore contributes to a band of convergence along the coast. This band of convergence acts to increase the temperature gradient, that is, to promote frontogenesis.

To summarize, the characteristics of coastal fronts have been shown to include the following:

- the presence of a shallow front separating cold continental air from warm maritime air;
- most common occurrence during the cold season (November -- March) and along concave coastlines (e.g., New England, Carolinas, Texas);
- quasistationary or warm-frontal structure and movement, often migrating inland with time;
- heaviest precipitation on cold side of front due to enhanced life there, especially with an extratropical cyclone located to the south;
- coastal fronts often form the eastern boundary of a cold-air damming event
- an "inverted trough" in the sea level pressure pattern can accompany the coastal front;
- coastal fronts can be accompanied by convection, even severe weather in some cases.
Chapter 5: Elementary Atmospheric Dynamics

The fluid atmosphere is a physical object and therefore, its motion is governed by the laws of physics. The purpose of this chapter is to describe the basic dynamics of the basic meteorological variables (i.e. pressure, temperature, humidity, and wind). We begin the discussion by describing the major forces that are active in the atmosphere.

5.1: Introduction

Newton’s second law states that the rate of change of momentum of an object (i.e. its acceleration) equals the sum of all the forces acting on that object:

\[
\frac{dp}{dt} = F_{\text{net}} \Rightarrow F_{\text{net}} = ma
\]

This powerful statement is valid only for motions measured in a non-accelerating coordinate system, i.e. one that is fixed in space (or moving at a constant velocity). Such a coordinate system is known as an inertial reference frame. The most convenient \(x, y,\) and \(z\) coordinates by which we measure motions on Earth refer to a grid based upon latitude and longitude (for the \(x\) and \(y\) coordinate directions) and elevation above sea level (for the \(z\) coordinate direction). Since the Earth rotates on its axis and revolves around the Sun, this Earth-based \((x, y, z)\) coordinate system undergoes constant acceleration. This fact is easily proven using a globe. After finding your location on the globe, consider the fact that what you view at that location as the immutable direction east is, in fact, constantly changing direction (to an observer fixed in space) as the Earth rotates on its axis. Thus our Earth-based coordinates are non-inertial (i.e. accelerating). This being the case, Newton’s second law can only be applied to the motion of objects on Earth if we correct for the acceleration of our coordinate system.

The collection of forces required to adequately represent Newton’s second law on the rotating Earth can therefore be split into two broad categories. The first of these includes forces that would affect objects even in the absence of rotation, the so-called fundamental forces. The most important of these fundamental forces are (1) the pressure gradient force, (2) the gravitational force, and (3) the frictional force, all of which we will investigate below. The other group of forces that we must consider in a full treatment of Newton’s second law arises from the
need to correct for the acceleration of our Earth-based coordinate system. We will refer to such forces as **apparent forces**. The two important apparent forces to be investigated in this chapter are (1) the centrifugal force and (2) the Coriolis force. We begin this examination by considering the nature of the pressure gradient force.

### 5.2: The Pressure Gradient Force

As can be readily discerned from inspection of any surface weather map, the pressure field varies zonally and meridionally. This spatial variability of pressure implies the presence of pressure gradients throughout the atmosphere. The **pressure gradient force** defines the force causing the air to respond to the pressure gradient.

![Figure 5.1 The x-component of the pressure gradient force acting on a fluid element](image)

In order to examine the pressure gradient force (PGF) we will consider the pressure exerted by the atmosphere on sides A and B of the air parcel illustrated in Figure 5.1. The pressure exerted on sides A and B arises from the fact that random molecular motions compel molecules to strike the sides. Each time a molecule strikes the side of the fluid element, a certain amount of momentum is transferred to that side (according to Newton's 2nd law). The total momentum transfer is the sum of all the individual momentum transfers from each molecule. Since \( dp/dt = F_{\text{net}} \), the total momentum transferred each second defines the force exerted by the atmosphere on the side of the fluid element. Since \( P = F_{\text{net}}/A \), dividing this total force by the area of the side of the fluid element defines the pressure that is exerted on that side.
The above considerations show that the magnitude of the PGF is largely proportional to the pressure gradient. It can be shown that the expression for the pressure gradient force (per unit mass) between two points can be written as

\[
\frac{F_{\text{pressure}}}{m} = -\frac{1}{\rho} \nabla p \approx -\frac{1}{\rho} \frac{P_1 - P_2}{D}
\]

where \( P_1 \) and \( P_2 \) are the pressure at the two points (here it's assumed that \( P_1 > P_2 \)) and \( D \) is the distance between the two points. The density prefactor, \( 1/\rho \), is included to account for the mass of the air. The negative sign in the above expression indicates that pressure gradient force is a vector that directed opposite of the pressure gradient. Physically, this states that the pressure gradient force compels air to move from regions of higher pressure to lower pressure. On surface weather charts, the above expression implies that the pressure gradient force is proportional to the spacing between isobars. Tightly packed isobars imply a strong pressure gradient force, whereas loosely packed isobars implies a weak pressure gradient force.

Figure 5.2: Sea-level pressure map at 12 Z on 18 December 2013 with wind vectors superimposed

Physically, the pressure gradient force demonstrates that any spatial variability of pressure in the atmosphere leads to the acceleration of air parcels. Since the change in pressure is directly proportional to the magnitude of pressure gradient force, this implies that the
acceleration of air parcels should increase as isobars become more closely spaced, as shown in Figure 5.2.

**5.2.1: Derivation of the Pressure Gradient Force**

Let’s assume that the air parcel in Figure 5.1 is an infinitesimal fluid element. If we define the pressure at the center of the fluid element to be \( p_0 \), then we can use a Taylor series expansion to determine the pressure on sides A and B

\[
P_A \approx p_0 + \frac{\partial p}{\partial x} \left( \frac{\delta x}{2} \right)
\]

\[
P_B \approx p_0 - \frac{\partial p}{\partial x} \left( \frac{\delta x}{2} \right)
\]

The x-direction pressure force on side A can be expressed as

\[
F_{Ax} = - \left( p_0 + \frac{\partial p}{\partial x} \delta x \right) y \delta z
\]

The x-direction pressure force on side B can be expressed as

\[
F_{Bx} = \left( p_0 - \frac{\partial p}{\partial x} \delta x \right) y \delta z
\]

Therefore, the net x-direction pressure force acting on the fluid element is

\[
F_{net, x} = F_{Ax} + F_{Bx} = - \frac{\partial p}{\partial x} \delta x y \delta z
\]

The net force per unit mass acting in the x-direction on the fluid element is

\[
\frac{F_x}{m} = - \frac{1}{\rho} \frac{\partial p}{\partial x}
\]

This procedure can be generalized to three dimensions to give

\[
\frac{\vec{F}_{\text{pressure}}}{m} = - \frac{1}{\rho} \vec{\nabla} p
\]
5.3: The Gravitational Force

Newton’s law of universal gravitation says that any two elements of mass in the universe attract each other with a force proportional to their masses and inversely proportional to the distance between their centers of mass. This is represented symbolically as

\[ \mathbf{F}_{\text{gravity}} = -G \frac{Mm \mathbf{r}}{r^2 \cdot r} \]

where \( G = 6.67 \times 10^{-11} \, \text{N m}^2 \text{kg}^{-2} \) is called the universal gravitational constant, \( \mathbf{r} \) is the position vector directed from the center of mass of \( M \) to the center of mass of \( m \). For an air parcel in the atmosphere, \( M \) is the mass of the Earth and \( m \) is the mass of an air parcel. Thus, we can express the gravitational force per unit mass as

\[ \frac{F_{\text{gravity}}}{m} = -G \frac{M \mathbf{r}}{r^2 \cdot r} \]

In the above expression, \( \mathbf{r} \) is the position vector directed from the center of the Earth to the center of mass of an air parcel in the atmosphere. Many applications in synoptic meteorology use height above sea level \( z \) as the vertical coordinate. This suggests that a parcel of air at a high elevation in the atmosphere might experience a smaller gravitational force than one located at sea level (i.e. nearer the center of gravity of the Earth). Though this is strictly true, the difference is very small from the surface to any level in the troposphere (lowest 10-12 km of the atmosphere) and we use a constant value of the gravitational force per unit mass, \( g^* \) where

\[ g^* = -G \frac{M \mathbf{r}}{a^2 \cdot r} \approx 9.80 \, \text{m/s}^2 \]

with \( a \) being the radius of the Earth, as a consequence.
5.4: The Frictional Force

Most of us have some conceptual understanding of friction and its effect on the behavior of solids. A textbook, for instance, that is pushed across a table feels the effect of the friction between itself and the tabletop and begins to decelerate immediately. In fact, the only reason the textbook does not continue to slide along the table indefinitely is that a force, the friction force, is applied opposite to its motion. The frictional force in this simple example is quantified as: \( f = \mu N \), where \( N \) is the normal force of the table and \( \mu \) is the coefficient of friction (which is a measure of the resistance to motion that results from pushing the book over the table).

This simplistic view of friction has to be modified when one considers the frictional force acting on a fluid parcel. Fluids, being collections of discrete atoms or molecules, are subject to internal friction (called viscosity) among these particles which cause the fluid to resist the tendency to flow. We will try to gain some insight into the nature of this resistance and how to express the physics in mathematical terms.

Consider, for instance, the situation depicted in Figure 5.3 in which a plate, moving at speed \( u_0 \), is placed atop a column of fluid with depth \( l \). The top layer of fluid moves at the velocity of the plate while the fluid at the bottom of the column has zero motion. Thus, a

![Figure 5.3: One dimensional steady-state viscous shear flow](image-url)
shearing stress exists in the fluid and a force must be exerted on the plate in order that it be kept moving at speed \( u_0 \) along the top surface of the fluid. The requisite force is proportional to \( u_0 \) since a greater force will be required for a greater speed. Additionally, since molecules of fluid that reside at the bottom of the column, can influence the movement of the plate through momentum transport in the fluid column, the requisite force is also inversely proportional to the depth of the fluid. The force is also proportional to the area of the plate since a larger plate makes contact with more fluid than a smaller one. With these considerations, the force required to keep the plate moving by dimensional analysis is given by

\[
F = \mu A \frac{du}{dz} \approx \mu A \frac{u_0}{l}
\]

where \( \mu \) is called the **dynamic viscosity coefficient.** The quantity \( du/dz \) is the change of velocity with height, also called the **vertical shear.** Physically, \( F \) represents the force required to overcome the viscous effect of the vertical shear. From the molecular viewpoint, a molecule moving to smaller \( z \) (i.e. toward the bottom of the fluid column) transports high momentum that it acquired from the motion of the plate to the surrounding fluid. Thus, there is a net downward transport of momentum and this momentum transport (per unit time) is the viscous force.

The above example can be applied to the atmosphere-Earth system where the bottom plate represents the surface of the Earth and the top plate represents the atmosphere. Based on the above analysis of our previous example, the effects of friction dominate near the surface where the momentum of air parcels is lost due to frictional dissipation with the surface. The region of the atmosphere where viscous forces become important is called the **boundary layer.** Observations of the mid-latitude synoptic environment suggest that top of the boundary layer is around 850 mb. As we will see later, surface friction will play a large role in the examination of the wind field.

**5.4.1: Derivation of the Viscous Force**

In the above expression, \( F \) represents the \( x \)-direction force required to overcome the viscous effect of the vertical shear of the \( x \)-direction velocity component. Hence, as \( \delta z \to 0 \), the shearing stress, or viscous force per unit area, is given by

\[
\tau_{zx} = \mu \frac{\partial u}{\partial z}
\]

where the subscript ‘\( zx \)’ indicates that this is the component of the shearing stress (in the \( x \) direction) that arises from the vertical shear (\( z \)) of the \( x \)-direction (\( x \)) velocity component. The prior example considered the steady movement of a plate across the top of a fluid column. In nature, viscous forces result from non-steady shear flows.
In recognition of this fact, let’s consider the volume element depicted in Figure 5.4, which represents the case of non-steady flow in a fluid of constant density. Analogous to our treatment of the pressure gradient force, we expand the shearing stress in a Taylor series in order to determine its value at the top and bottom (z-direction) facing sides of the volume element. The stress acting across the upper boundary on the fluid below it can be approximated as

$$\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}$$

while the stress acting across the bottom boundary on the fluid below it can be approximated as

$$\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}$$

According to Newton’s 3rd law, this stress must be equal and opposite to the stress acting across the bottom boundary on the fluid above it. Since we are interested in the net stress acting on the volume element in Figure 5.4, we want to sum the forces that act on fluid within the volume element. Thus, we find that the net viscous force on the volume element acting in the x-direction is given by

$$\left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}\right) \delta x \delta y - \left(\tau_{zx} - \frac{\partial \tau_{zx}}{\partial z} \frac{\delta z}{2}\right) \delta x \delta y = \frac{\partial \tau_{zx}}{\partial z} \delta x \delta y \delta z$$

Dividing this expression by the mass of volume element, \(\rho \delta x \delta y \delta z\), we have the viscous force per unit mass arising from the vertical shear of the x-direction motion:

$$\frac{1}{\rho} \frac{\partial \tau_{zx}}{\partial z} = \frac{1}{\rho} \frac{\partial}{\partial z} \left(\mu \frac{\partial u}{\partial z}\right)$$
If $\mu$ is constant, then the above expression can be reduced to

$$\frac{1}{\rho} \frac{\partial}{\partial z} \left( \mu \frac{\partial u}{\partial z} \right) = \nu \frac{\partial^2 u}{\partial z^2}$$

where $\nu = \mu/\rho$ is known as the **kinematic viscosity coefficient**. Analogous derivations can be performed to determine the viscous stresses acting in the other directions. The resulting frictional force per unit mass in the $x$, $y$, and $z$ directions is

$$\bar{F}_{fr} = \nu \nabla^2 \vec{v}$$

### 5.5: Introduction to the Apparent Forces

In expressing his first law, Sir Isaac Newton states: “Every body persists in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed on it.” In other words, a mass in uniform motion relative to a coordinate system fixed in space will remain in uniform motion in the absence of any forces. Any motion relative to a coordinate system fixed in space is known as **inertial motion** and the reference frame in which that motion is measured is known as an **inertial reference frame**. Most of us live at a single location long enough to become accustomed to thinking of north, south, east, and west as fixed directions. In reality, however, the direction I call “north” is not the same, as viewed from the perspective of a space traveler orbiting Earth. If one considers the intersection of latitude and longitude lines on a globe as the intersections of a Cartesian $x$ and $y$ grid describing the Earth, then it is clear that since the Earth rotates, this coordinate system is accelerating and thus provides us with a non-inertial reference frame. It might appear that given our non-inertial reference frame we are not able to apply Newton's laws of motion to motion relative to the Earth. Of course, this is not true, but we do have to make some correction for the non-inertial nature of the reference frame by which we measure all such motion. We will make the necessary corrections by introducing the **centrifugal** and **Coriolis forces**, the so-called **apparent forces**. But first, it is instructive to consider physically why the coordinate system matters at all. We can do this by considering application of Newton's laws to experiments conducted inside a closed elevator car.

In the first case, let us imagine that the car is stationary or moving with a constant velocity, $\vec{V}$. Under such conditions imagine that a weight is dropped within the moving car. Upon
making the appropriate measurements and calculations, you would determine that the weight had fallen toward the floor of the car with a measurable, constant acceleration of $9.80 \text{ m/s}^2$. This acceleration would be observed relative to the walls and floor of the elevator car in a Cartesian coordinate system defined by the dimensions of the elevator car. In such a case, an observer in the elevator car would note complete agreement between the results of the experiment and Newton's laws of motion since the constant velocity elevator car provides an inertial reference frame for this experiment.

In the second case, we remotely observe the elevator car falling freely through the elevator shaft. If a similar weight is dropped within the car the weight appears to remain suspended in mid-air, at a constant elevation above the floor of the car. Measured relative to the coordinate frame of the car, the weight has zero acceleration even though to us remote observers it is clearly accelerating toward the ground at a rate of $9.80 \text{ m/s}^2$. Viewed from inside the car, Newton’s laws seem to fail here, but this is because the coordinate system itself is accelerating and is therefore non-inertial. The latitude/longitude coordinate system on a rotating Earth is also accelerating and so we have to take that acceleration into account in order to apply Newton’s laws accurately to objects moving relative to that Earth-based coordinate system.

5.6: The Centrifugal Force

Figure 5.5 The rotating ball on a string experiences an inward-directed centripetal acceleration, indicated by the red arrow. To the observer on the ball, a compensating centrifugal force, indicated by the purple arrow, must be included to describe accurately motions on the ball itself.

Each of us is located a certain distance from the axis of rotation of the Earth. Depending upon the exact distance, we are rotating around that axis at a very high speed, but constant speed. Each of us is, therefore, not unlike the ball on the end of the string depicted in
The speed of the ball is \( v = \omega r \), where \( \omega \) is angular speed of the ball and \( r \) is the radius of rotation. However, the direction of the ball changes continuously and so, as viewed from the perspective of the ball, there is a uniform centripetal acceleration caused by the force of the string pulling on the ball directed toward the axis of rotation equal to

\[
\frac{dv}{dt} = -\frac{v^2}{r} = -\omega^2 r
\]

Suppose you are on the ball and rotating with it. From your perspective the ball is stationary but, in reality, a centripetal acceleration is still being exerted upon it. In order for a person on the ball to apply Newton’s laws under this condition, an apparent force that exactly balances the true centripetal force must be included in the physics; this apparent force is known as the centrifugal force. In order to balance the centripetal acceleration, the centrifugal acceleration is directed outward along the radius of rotation and is given by \( \omega^2 r \).

As depicted in Figure 5.6, on a rotating Earth, the centrifugal force affects the vertical force balance. When the centrifugal force and gravitational force \( \vec{g}' \) are added, the result is called effective gravity (\( \vec{g} \)) and is given by \( \vec{g} = \vec{g}' + \Omega^2 \vec{R} \) where \( \Omega \) is the rotation rate of the Earth, and \( \vec{R} \) is the position vector from the axis of rotation to the object in question. Note that effective gravity, thus defined, is directed perpendicular to the local tangent of the surface of the Earth -- not necessarily toward the center of the Earth. In fact, since \( \Omega^2 \vec{R} \) is directed away from the axis of rotation, \( \vec{g} \) is not directed toward the center of the Earth except at the poles and the equator! Were the Earth a perfect sphere, this fact would result in the existence of a horizontal,
The equatorward-directed component of gravity. The relatively malleable crust of the Earth has long since responded to this circumstance and adopted its oblate spheroidal shape with an equatorial radius some 21 km larger that its polar radius. Given such a slightly distorted shape, the local vertical direction everywhere on Earth is defined parallel to $\vec{g}$. The centrifugal force component of effective gravity is an example of the effect of rotation on objects at rest with respect to the Earth-based rotating frame of reference. In order to apply Newton’s laws accurately to the motion of objects relative to that rotating frame an additional apparent force, the Coriolis force, must be considered.

### 5.7: The Coriolis Force

Consider a experiment in which one student takes a position on a merry-go-round and another student takes a position some distance above the ground in an adjacent tree. The merry-go-round is set spinning and a ball is pushed from the center of the merry-go-round toward the spinning student. From the vantage point of the tree, the motion of the ball appears as a straight
line, as it should since a uniform force was administered to it. But from the perspective of the rotating frame, the ball appears to accelerate in a curved path, away from the observer in a direction opposite to the direction of rotation. Upon consulting each other’s notes, the students conclude that an apparent force, arising from the rotation of the merry-go-round, deflects the ball from its path. This apparent force is the Coriolis force.

A careful examination of our thought experiment above reveals more information and details about the Coriolis force.

- The magnitude of the Coriolis force depends on the rotation rate of the Earth. The deflection of the ball from its path would increase if the rotation rate of the merry-go-round increased.
- The magnitude of the Coriolis force depends on latitude. The deflection of the ball from its path is felt more at the rim of the merry-go-round than the center of the merry-go-round. On the Earth, the Coriolis force is zero at the equator and increases towards the poles.
- The magnitude of the Coriolis force is proportional to the velocity of the object. If the object's speed is zero, there is no relative motion and thus the Coriolis force is zero. Since an object's momentum increases with speed, a larger force is required to change its momentum. This suggests that the Coriolis force increases as the velocity of the object increases.
- The Coriolis force always acts to deflect an object to the right (left) of its direction of motion in the Northern (Southern) hemisphere (as shown in Figure 5.8). This also implies that the Coriolis force acts perpendicular to the direction of motion. Since the Coriolis force always acts perpendicular to the motion vector, it can do no work on the moving object. Thus, the Coriolis force can only change the direction of motion but cannot initiate motion in an object at rest.
These considerations suggest that the magnitude of the Coriolis force depends upon wind speed, the rotation rate of the Earth, and the latitude. It can be shown that the magnitude of the Coriolis force is given by

\[ F_{\text{cor}} = 2\Omega |\vec{V}| \sin \phi \]

where \( \Omega \) is the rotation rate of the Earth and \( \phi \) is the latitude. Using a shorthand in which \( f \), the so-called Coriolis parameter, is given by \( f = 2\Omega \sin \phi \), we can rewrite the magnitude of the Coriolis force as \( F_{\text{cor}} = f |\vec{V}| \). Because of the Coriolis force, an object moving eastward (westward) will deflect to the south (north) in the Northern Hemisphere. Similarly, an object moving northward (southward) will deflect to the east (west) in the Northern Hemisphere.
**5.7.1: Derivation of Apparent Forces**

To derive the apparent forces, let the $X$ and $Y$ axes form the inertial reference frame and the $x$ and $y$ axes be those of a rotating reference frame (with a rotation rate $\Omega$) with the same origin as shown in Figure 5.9. The corresponding unit vectors are denoted $(\mathbf{i}, \mathbf{j})$ and $(\mathbf{I}, \mathbf{J})$. At any time $t$, the rotating $x$-axis makes an angle $\Omega t$ with the fixed $X$-axis. It follows that

\begin{align*}
\mathbf{i} &= \mathbf{I} \cos \Omega t + \mathbf{J} \sin \Omega t, \quad \mathbf{I} = \mathbf{i} \cos \Omega t - \mathbf{j} \sin \Omega t \\
\mathbf{j} &= -\mathbf{I} \sin \Omega t + \mathbf{J} \cos \Omega t, \quad \mathbf{J} = \mathbf{i} \sin \Omega t - \mathbf{j} \cos \Omega t
\end{align*}

and that the coordinates of the position vector $\mathbf{r} = XI + YJ = xi + yj$ of any point in the plane are related by

\begin{align*}
x &= X \cos \Omega t + Y \sin \Omega t, \quad y = -X \sin \Omega t + Y \cos \Omega t
\end{align*}

The first time derivative of the above expression gives

\begin{align*}
\frac{dx}{dt} &= \frac{dX}{dt} \cos \Omega t + \frac{dY}{dt} \sin \Omega t - \Omega X \sin \Omega t + \Omega Y \cos \Omega t = \frac{dX}{dt} \cos \Omega t + \frac{dY}{dt} \sin \Omega t + \Omega y \\
\frac{dy}{dt} &= -\frac{dX}{dt} \sin \Omega t + \frac{dY}{dt} \cos \Omega t - \Omega X \cos \Omega t - \Omega Y \sin \Omega t = -\frac{dX}{dt} \sin \Omega t + \frac{dY}{dt} \cos \Omega t - \Omega x
\end{align*}

A second derivative with respect to time provides in a similar manner
The relative acceleration (in the rotating frame) is given by
\[
\frac{d^2x}{dt^2} = \left( \frac{d^2X}{dt^2} \cos \Omega t + \frac{d^2Y}{dt^2} \sin \Omega t \right) + 2\Omega \left( -\frac{dX}{dt} \sin \Omega t + \frac{dY}{dt} \cos \Omega t \right)
- \Omega^2(X \cos \Omega t + Y \sin \Omega t) = \left( \frac{d^2X}{dt^2} \cos \Omega t + \frac{d^2Y}{dt^2} \sin \Omega t \right) + 2\Omega V - \Omega^2 x
\]
\[
\frac{d^2y}{dt^2} = \left( -\frac{d^2X}{dt^2} \sin \Omega t + \frac{d^2Y}{dt^2} \cos \Omega t \right) + 2\Omega \left( \frac{dX}{dt} \cos \Omega t + \frac{dY}{dt} \sin \Omega t \right)
- \Omega^2(-X \sin \Omega t + Y \cos \Omega t) = \left( -\frac{d^2X}{dt^2} \sin \Omega t + \frac{d^2Y}{dt^2} \cos \Omega t \right) - 2\Omega U - \Omega^2 y
\]

The relative acceleration (in the rotating frame) is given by
\[
\vec{a} = \frac{d^2x}{dt^2} \hat{i} + \frac{d^2y}{dt^2} \hat{j}
\]

The absolute acceleration (in the inertial frame) is given by
\[
\vec{A} = \frac{d^2X}{dt^2} \hat{i} + \frac{d^2Y}{dt^2} \hat{j} = \left( \frac{d^2X}{dt^2} \cos \Omega t + \frac{d^2Y}{dt^2} \sin \Omega t \right) \hat{i} + \left( -\frac{d^2X}{dt^2} \sin \Omega t + \frac{d^2Y}{dt^2} \cos \Omega t \right) \hat{j}
\]

Comparing these expressions gives
\[
A_x = a_x - 2\Omega v - \Omega^2 x, \quad A_y = a_y + 2\Omega u - \Omega^2 y
\]

We now see that the difference between absolute and relative acceleration consists of two contributions. The first contribution, proportional to \(\Omega\), is called the **Coriolis acceleration** and the second contribution, proportional to \(\Omega^2\), is called the **centrifugal acceleration**. The preceding results can also be written in vector form. Defining \(\vec{\Omega} = \Omega \hat{k}\) where \(\hat{k}\) is the unit vector in the third dimension, we have \(\vec{A} = \vec{a} + 2\vec{\Omega} \times \vec{u} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})\). We now see that
\[
\vec{F}_{cor} = 2\vec{\Omega} \times \vec{u}, \quad \vec{F}_{cen} = \vec{\Omega} \times (\vec{\Omega} \times \vec{r})
\]
We have now considered all the forces necessary to accurately describe the dynamics of an air parcel in the atmosphere. We are now in the position to derive the fundamental equations of motion.

**5.8: The Fundamental Equations of Motion**

Recall that Newton’s 2nd law is a statement of the conservation of momentum:

\[ \frac{d}{dt}(m\vec{V}) = \vec{F}_{net} \]

However, it is strictly true, as we have already considered, only in an inertial frame of reference. As discussed previously, the five major forces that impact atmospheric motion are the pressure gradient force, the gravitational force, the frictional force, the centrifugal force, and the Coriolis force. This implies the acceleration of an air parcel following the relative motion in a rotating reference frame is equal to the sum of the atmospheric forces.

From section 5.7.1, we derived a relationship between the absolute acceleration of an air parcel \( \vec{A} \) and the acceleration of the same air parcel relative to Earth \( \vec{a} \). Defining \( \vec{A} = \frac{d_a\vec{V}_a}{dt} \) and \( \vec{a} = \frac{d\vec{V}}{dt} \), we have

\[
\frac{d_a\vec{V}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r})
\]

In words, the above expression states that the Lagrangian acceleration in an inertial system is equal to the sum of (1) the Lagrangian change of relative velocity, plus (2) the Coriolis acceleration from relative motion in the relative frame, plus (3) the centripetal acceleration resulting from the rotation of the coordinate system. Recalling Newton’s second law and the fact that we will consider the pressure gradient force, the frictional force, and gravitational force as the only real forces acting on the atmospheric fluid, we find that

\[
\frac{d_a\vec{V}_a}{dt} = \frac{d\vec{V}}{dt} + 2\vec{\Omega} \times \vec{V} + \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = -\frac{1}{\rho} \nabla p + \vec{g}^* + \nu \nabla^2 \vec{V}
\]

Rearranging in terms of the relative acceleration gives

\[
\frac{d\vec{V}}{dt} = -\frac{1}{\rho} \nabla p - 2\vec{\Omega} \times \vec{V} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) + \vec{g}^* + \nu \nabla^2 \vec{V} = -\frac{1}{\rho} \nabla p + \vec{g} + \nu \nabla^2 \vec{V}
\]

where the centripetal force has been combined with the gravitation force \( \vec{g}^* \) in the gravity term \( \vec{g} \). This expression states that the acceleration following the relative motion in a rotating reference frame is equal to the sum of (1) the Coriolis force, (2) the pressure gradient force, (3) effective gravity, and (4) the friction force. This is a major result, but it remains in vectorial form.
only – a form not particularly amenable to analysis. Since synoptic-scale weather systems of interest are sufficiently small relative to the size of the earth, it is acceptable to write the vector momentum equation in terms of Cartesian coordinates.

First, the component form of the pressure gradient force is given by

\[- \frac{1}{\rho} \nabla p = - \frac{1}{\rho} \frac{\partial p}{\partial x} \hat{i} - \frac{1}{\rho} \frac{\partial p}{\partial y} \hat{j} - \frac{1}{\rho} \frac{\partial p}{\partial z} \hat{k}\]

The effective gravity, which acts downward in the local vertical direction, is represented by

\[\vec{g} = -g \hat{k}\]

while friction can be represented as

\[\nu \nabla^2 \vec{V} = \nu \nabla^2 u \hat{i} + \nu \nabla^2 v \hat{j} + \nu \nabla^2 w \hat{k}\]

Figure 5.10 Partition of the rotation vector $\vec{\Omega}$ into its vertical and meridional components

Figure 5.10 demonstrates that the rotation vector $\vec{\Omega}$ is perpendicular to the $x$ direction and so has components only in the positive $\hat{j}$ and positive $\hat{k}$ directions. Considering the trigonometry in Figure 5.10, the $\hat{k}$ component of $\vec{\Omega}$ has magnitude $\Omega \sin \phi$ while the $\hat{j}$ component has magnitude $\Omega \cos \phi$. Thus, the components of the Coriolis force term is given by

\[-2\Omega \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2\Omega \cos \phi & -2\Omega \sin \phi \\ u & v & w \end{vmatrix} = -(2\Omega \cos \phi w - 2\Omega \sin \phi v) \hat{i} - 2\Omega \sin \phi u \hat{j} + 2\Omega \cos \phi u \hat{k}\]
Therefore, the three component equations of motion for flow on the rotating Earth is given by

\[
\frac{du}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + 2\Omega \sin \phi v - 2\Omega \cos \phi w + \nu \nabla^2 u
\]

\[
\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - 2\Omega \sin \phi u + \nu \nabla^2 v
\]

\[
\frac{dw}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + 2\Omega \cos \phi u + \nu \nabla^2 u
\]

In the next chapter, we will use these equations to develop an understanding of synoptic-scale motions.
Chapter 6: The Governing Equations

In the previous chapter, we considered all the forces necessary to accurately describe the dynamics of an air parcel in the atmosphere. This led us to the fundamental equations of motion for the atmosphere. In this chapter, we will use these equations of motion to develop a practical, dynamical understanding of the atmosphere at the middle latitudes.

**6.1: Introducing Scale Analysis

The complete set of governing equations for the atmosphere is difficult to utilize and conceptually comprehend. For specific applications, the equations can be simplified through the elimination of terms that are unimportant to the situation in question. A procedure known as scale analysis is a systematic strategy to determine which terms in the equations, often associated with specific physical processes, are most important and which are negligible in a given meteorological setting. By characterizing the temporal and spatial scales associated with specific weather systems, we can systematically neglect “small” terms in the governing equations in the study of those systems. While these assumptions reduce accuracy, their use is justified by the physical insight obtained from the simplified equations.

Why is it important to derive customized equation sets and techniques for different classes of weather system, and why must students work through these derivations? In addition to the dynamical insight afforded by isolating the essential physics of a given weather system, one must know when to apply and when not to apply a given technique. Suppose that a particular weather forecasting technique is developed from simplified equations that are valid for synoptic-scale flows, and that this technique gains widespread acceptance in the forecasting community. If a forecaster were to apply this technique to a mesoscale weather system, the technique may fail, perhaps resulting in a poor weather forecast. This is one example of why students are required to derive equations, because it is important to know what assumptions were made in the development of a given technique, and this information is needed to deduce which tools are appropriate for which situations.

In addition, the ability to apply a systematic approach to the governing equations allows atmospheric scientists to develop new equations and techniques to study unique problems. In applying scale analysis to various weather systems, we must identify the characteristic horizontal length and time scales, and these are often related to one another. The length scale can be related to the size of a weather system, or how far an air parcel would travel within the system during a
given time interval. The time scale can be related to how long it would take an air parcel to circulate within the system, or to traverse the characteristic length scale (implying a characteristic velocity scale).

**6.1.1: Simplifying the Equations of Motion**

The observationally based characteristic values for the set of variables in the equations of motion are given by

\[
\begin{align*}
U &\approx 10 \text{ m/s} \quad \text{Characteristic horizontal velocity} \\
W &\approx 1 \text{ cm/s} \quad \text{Characteristic vertical velocity} \\
L &\approx 1000 \text{ km} = 10^6 \text{ m} \quad \text{Characteristic length scale of synoptic-scale features} \\
H &\approx 10 \text{ km} = 10^4 \text{ m} \quad \text{Characteristic depth scale} \\
\delta p &\approx 10 \text{ mb} = 1000 \text{ Pa} \quad \text{Characteristic horizontal pressure fluctuation} \\
P_0 &\approx 1000 \text{ mb} = 10^5 \text{ Pa} \quad \text{Characteristic pressure} \\
t &\approx L/U = 10^5 \text{ s} \quad \text{Characteristic time scale}
\end{align*}
\]

We can now estimate the magnitude of each term in the equations of motion for a given latitude. For the midlatitudes, it is convenient to consider a disturbance centered at latitude \(\phi_0 = 45^\circ\) and introduce the notation

\[
f_0 = 2\Omega \sin \phi_0 = 2\Omega \cos \phi_0 \approx 10^{-4} \text{ s}^{-1}
\]

Figure 6.1 shows the characteristic magnitude of each term in the horizontal equations of motion based on the scaling considerations given above. The molecular friction term is so small that it may be neglected for all motions except the smallest scale turbulent motions near the ground. It is apparent from Figure 6.1 that for midlatitude synoptic scale disturbances the Coriolis force (term B) and the pressure gradient force (term F) are in approximate balance. Retaining only these two terms gives as a useful simplification of the horizontal equations of motion

\[
-fv \approx -\frac{1}{\rho} \frac{\partial p}{\partial x}; \quad fu \approx -\frac{1}{\rho} \frac{\partial p}{\partial y}
\]
where $f \equiv 2\Omega \sin \phi$ is called the *Coriolis parameter*. This balance is known as **geostrophic balance**.

### Scale Analysis of the Horizontal Momentum Equations

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$-Eq.</td>
<td>$\frac{Du}{Dt}$</td>
<td>$-2\Omega u \sin \phi$</td>
<td>$+2\Omega u \cos \phi$</td>
<td>$+\frac{u w}{a}$</td>
<td>$-\frac{u w \tan \phi}{a}$</td>
<td>$=-\frac{1}{\rho} \frac{\partial p}{\partial z}$</td>
<td>$+F_{zx}$</td>
</tr>
<tr>
<td>$y$-Eq.</td>
<td>$\frac{Dv}{Dt}$</td>
<td>$+2\Omega u \sin \phi$</td>
<td>$+\frac{v w}{a}$</td>
<td>$+\frac{v^2 \tan \phi}{a}$</td>
<td>$=-\frac{1}{\rho} \frac{\partial p}{\partial z}$</td>
<td>$+F_{zy}$</td>
<td></td>
</tr>
<tr>
<td>Scales</td>
<td>$U^2/L$</td>
<td>$f_0U$</td>
<td>$f_0W$</td>
<td>$\frac{U W}{a}$</td>
<td>$\frac{U^2}{a}$</td>
<td>$\frac{\delta P}{\rho L}$</td>
<td>$\frac{v U}{H^2}$</td>
</tr>
<tr>
<td>$(\text{m s}^{-2})$</td>
<td>$10^{-4}$</td>
<td>$10^{-3}$</td>
<td>$10^{-6}$</td>
<td>$10^{-8}$</td>
<td>$10^{-5}$</td>
<td>$10^{-3}$</td>
<td>$10^{-12}$</td>
</tr>
</tbody>
</table>

### Scale Analysis of the Vertical Momentum Equation

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$-Eq.</td>
<td>$\frac{Dw}{Dt}$</td>
<td>$-2\Omega u \cos \phi$</td>
<td>$-(u^2 + v^2)/a$</td>
<td>$=-\rho^{-1} \frac{\partial p}{\partial z}$</td>
<td>$-g$</td>
<td>$+F_{rz}$</td>
<td></td>
</tr>
<tr>
<td>Scales</td>
<td>$U W/L$</td>
<td>$f_0U$</td>
<td>$U^2/a$</td>
<td>$P_0/(\rho H)$</td>
<td>$g$</td>
<td>$v WH^{-2}$</td>
<td></td>
</tr>
<tr>
<td>$(\text{m s}^{-2})$</td>
<td>$10^{-7}$</td>
<td>$10^{-3}$</td>
<td>$10^{-5}$</td>
<td>$10$</td>
<td>$10$</td>
<td>$10^{-15}$</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6.1: Scale analysis of the equations of motion

A similar scale analysis can be applied to the vertical component of the momentum equation. Because pressure decreases by about an order of magnitude from the ground to the tropopause, the vertical pressure gradient may be scaled by $P_0/H$, where $P_0$ is the surface pressure and $H$ is the depth of the troposphere. The terms in the vertical momentum equation are also given in Figure 6.1. As with the horizontal momentum equations, we consider motions centered at 45° latitude and neglect friction. The scaling indicates that to a high degree of accuracy, the gravitational force and the pressure gradient force are in approximate balance. Retaining only these two terms gives as a useful simplification of the vertical momentum equation:

$$-g \approx -\frac{1}{\rho} \frac{\partial p}{\partial z}$$

This balance is known as **hydrostatic balance**.

Thus, an analysis of the momentum equations for mid-latitude synoptic-scale motions demonstrate that the vertical equation of motion is dominated by two terms, the vertical pressure gradient force and the gravitational force, and the horizontal equations of motion are dominated by two terms, the horizontal pressure gradient force and the Coriolis force. The first condition
leads to hydrostatic balance, whereas the second condition leads to a balanced condition called geostrophic balance. Thus, a scaling of the equations of motion for mid-latitude synoptic scale motions renders the following fundamental statement regarding the nature of the mid-latitude atmosphere on Earth:

**To a first order, the mid-latitude atmosphere on Earth is in hydrostatic and geostrophic balance**

### 6.2: The Hydrostatic Equation

Apart from performing a scaling analysis, another important way to describe the physical behavior of the atmosphere is to examine the fundamental conservative quantities that govern the atmosphere: mass, momentum, and energy. In the next sections, we will investigate the manner in which these quantities and their various interactions serve to describe the building blocks of a dynamical understanding of the atmosphere at the middle latitudes. The first quantity that we will study will be mass. For our purposes, we shall define mass as the measure of the substance of an object and make that measurement in kilograms (kg). We must first consider the distribution of mass in the atmosphere and the force balance that underlies this distribution. A number of insights concerning the vertical structure of the atmosphere proceed directly from this understanding.

![Figure 6.2 Schematic of the distribution of mass in the atmosphere](image)
Observations show that the pressure decreases exponentially with height within the troposphere, which implies that most of the mass of the atmosphere resides near Earth's surface. As a consequence, there is a strong vertical pressure gradient force that exists throughout the atmosphere which compels air to move from higher pressure (near the surface) to lower pressure (above the surface) (i.e. the vertical pressure gradient force is an upward-directed force). The fact that the atmosphere does not race away into space under this forcing is a consequence of the fact that there is also the force of gravity acting on the atmosphere, pulling it downwards.

The atmosphere tends to exhibit a balance between this vertical pressure gradient force, which points upward, and the gravitational force, which points downward. We call this balance of forces hydrostatic balance. The condition for hydrostatic balance leads to the following expression:

\[
\frac{dP}{dz} = -\rho g
\]

This expression, known as the hydrostatic equation, represents a fundamental balance characteristic of the Earth's atmosphere. Though strictly true only for an atmosphere at rest (hence the static portion of the name), this hydrostatic balance is obeyed to great accuracy under nearly all conditions in the Earth’s atmosphere.

### 6.3: The Thickness Equation

Consider the unit area column of atmosphere contained between 850 hPa and 500 hPa shown in Figure 6.3. Since pressure is defined as force per unit area, we have isolated in that column an atmospheric mass sufficient to exert 350 hPa of pressure. Such a slab of the atmosphere has a unique mass whether it extends from 850 to 500 hPa or from 717 to 367 hPa. In fact, the mass of this column can be precisely calculated as 3567.79 kg. Though the mass of a 350 hPa, unit area slab of the atmosphere is unique, its depth might be different from one day to the next. We will refer to this geometric depth as the thickness between two isobaric surfaces.
Clearly, if the thickness varies, then so does the volume of the unit area slab. The variation of the volume of the slab dictates that the density of the air contained within the slab varies as well: less (more) dense air will correspond to a greater (smaller) thickness. By the ideal gas law, less (more) dense air will correspond to a higher (lower) column average virtual temperature, $T_v$. Thus, column average virtual temperature should have a bearing on the thickness between two isobaric levels.

Combining the hydrostatic equation with the ideal gas law provides convincing evidence to support this hypothesis. Recall that the ideal gas law can be written as $P = \rho R_d T_v$ where $P$ is the pressure, $\rho$ is the density, $R_d$ is the gas constant for dry air, and $T_v$ is the virtual temperature. Using this expression, the hydrostatic equation can be rewritten as

$$\frac{dP}{dz} = -\frac{\rho g}{R_d T_v} \Rightarrow dz = -\frac{R_d T_v}{g} \frac{dP}{P}$$

If we integrate this expression between pressure levels $P_1$ and $P_2$ at which the heights are $z_1$ and $z_2$, we get

$$\frac{R_d T_v}{g} \ln \left( \frac{P_1}{P_2} \right) = z_2 - z_1 = \Delta z$$
where $\bar{T}_v$ is the pressure-weighted, column average virtual temperature. This expression is known as the **thickness equation** and it quantifies our suspicion regarding the influence of column average temperature on the thickness of an isobaric column.

We can express the thickness equation (and, therefore, the hydrostatic equation) in terms of a quantity called **geopotential**, $\Phi$. The geopotential is defined as the work required to raise a unit mass a distance $dz$ above sea level. It quantifies the work (per unit mass) that is done against gravity in doing so. Mathematically, therefore, geopotential is given as $d\Phi = g dz$. Using this expression, we can rewrite the hydrostatic equation as

$$\frac{d\Phi}{dP} = -\frac{R_d \bar{T}_v}{P}$$

Correspondingly, the thickness equation can also be written as

$$R_d \bar{T}_v \ln \left( \frac{P_1}{P_2} \right) = g \Delta z = \Delta \Phi$$

Writing the geopotential in units of height gives us a new meteorology variable called **geopotential height**. The geopotential height is simply given by $Z = \Phi / g_0$ where $g_0$ is the global average gravity at sea level (9.80 m/s²). Thus, geometric height ($z$) and geopotential height $Z$ are just about equal in the troposphere. Because geopotential height includes the variation of gravity with elevation, the geopotential height is sometimes referred to as a “gravity-adjusted height”.

### 6.3.1: Reduction to Sea Level Pressure

There are several important applications of the hydrostatic and thickness equations that have a bearing on the analysis and understanding of mid-latitude weather systems. One of the most common analysis products used to characterize and understand the weather is a sea level pressure map (as shown in Figure 6.5). This is a map on which isobars of sea-level pressure are contoured in an attempt to identify and characterize the major circulation systems in a given location at a given time. In geographical regions characterized by high terrain, such as the Rocky Mountains of North America, the elevation is so far above sea level that use of the station pressure (i.e. the pressure actually measured with a barometer at the station) does not effectively
contribute to this goal. In such regions the thickness equation can be used to calculate a **reduced sea-level pressure** (i.e. an estimate of what the sea-level pressure would be were the surface elevation 0 m as shown in Figure 6.4). Consider the following example.

![Figure 6.4 Schematic of sea-level pressure chart](image)

In Figure 6.5, the station pressure at St Louis, Missouri (STL), a city close to sea level, on a certain day is measured to be 1010 hPa. Meanwhile, the station pressure at Denver, Colorado (DEN), whose elevation is 1609 m above sea level, is measured at 828 hPa. There is not a horizontal pressure difference of 180 hPa between STL and DEN. Most of the observed pressure difference is a consequence of the vertical variation of pressure. By reducing the station pressure to sea level at DEN, we attempt to discover how much of the observed pressure difference actually is a horizontal pressure difference.
We begin with the thickness equation \( z_2 - z_1 = (R_d \bar{T}_v/g) \ln(P_1/P_2) \) with \( z_2 = z_{DEN} \) and \( z_1 = 0 \) (the geometric height at sea level). Correspondingly, \( P_2 = P_{STA} \) (observed station pressure) and \( P_1 = P_{SLP} \) (the desired value we calculate as sea level pressure at DEN). Finally, \( \bar{T}_v \) represents the average column temperature between sea level at DEN and the station elevation. Rearranging the thickness equation using the given variables, we have

\[
\frac{gz_{DEN}}{R_d \bar{T}_v} = \ln \left( \frac{P_{SLP}}{P_{STA}} \right) \Rightarrow P_{SLP} = P_{STA} \exp \left[ \frac{gz_{DEN}}{R_d \bar{T}_v} \right]
\]

The above expression is known as the **altimeter equation** and is the standard expression for reducing station pressure to sea level. Supposing that the surface \( T_v \) at Denver is 20°C, we find that the reduced sea-level pressure at Denver would be 1012.5 hPa. This value can be usefully compared to the sea-level pressure at St. Louis on a synoptic weather chart.
6.3.2: Isobaric Charts

The definition of geopotential height and the thickness equation enables meteorologists to construct upper-air maps at constant pressure, also known as isobaric charts (as shown in Figure 5.5). By convention, radiosonde data records and reports upper air observations at 1000, 925, 850, 700, 500, 400, 300, 250, 200, 150, and 100 mb. Since pressure is fixed on isobaric charts, we can use the thickness equation to examine how geopotential height varies across that surface.

Figure 6.5 850-mb map analysis of the continental U.S. on 01 April 2014 at 1200 UTC. Black contours are geopotential height lines, red contours are isotherms greater than 0°C, and blue contours are isotherms less than 0°C.
Since the atmospheric depth (i.e. thickness) is proportional to the mean atmospheric temperature, the isobaric surfaces will be located at higher level further south and at lower levels further north (as shown in Figure 6.6). Therefore, on an isobaric map, we plot lines of constant geopotential height (also called isohypses). Because of the relationship between pressure and height, the kinematics of pressure contours on a surface map is identical to the kinematics of geopotential height contours on upper-air isobaric charts. Thus, regions of low isohypse values are correlated with low pressure (trough) while isohypse values are correlated with high pressure (ridges).

6.3.3: Thickness and Temperature Advection

As mentioned previously, the thickness equation allows us to make a general relationship between temperature and geopotential heights on isobaric maps. As a rule, warmer temperatures in the lower troposphere imply higher geopotential heights at upper levels. Since warm air leads to a greater thickness than cold air, thickness is another easy way to diagnose where warm and cold air masses are present in the atmosphere. This also implies that thickness analyses can be used to locate synoptic fronts and to determine their intensity (since by definition a synoptic front must be associated with a horizontal thickness gradient). Consider the 500 mb map shown in Figure 6.7. Note that the regions of lowest thickness also correspond to the lowest geopotential heights. This means that a cold air mass is present in NE Canada and the wind field is advecting cold, polar air towards the northeast and Midwest US.
Figure 6.7 1000-500 hPa thickness and geopotential height map from 29 January 2014 at 06 Z

Thickness is often used to determine the 50% probability of snow, given that precipitation is occurring. This is referred to as the rain-snow line. Figure 5.8 shows thickness values for the 1000-500 mb layer associated with a 50% chance of snow given that precipitation is occurring. Comparing Figure 6.8 to Figure 6.7, it seems likely that if precipitation is occurring in the Midwest, then the 1000-500 mb layer indicates snow as the more probable form of precipitation. In the eastern US near sea level, ΔZ = 5400 m for the 1000 to 500 mb layer (with mean virtual temperature of -7.1°C) closely corresponds to the 50% probability between rain and snow if precipitation is occurring. Consequently, the 5400 m line is used as an indicator of the rain-snow line. Note that the rain-snow line is substantially higher for sites at relatively high altitudes like Denver and Cheyenne since the warmest part of the layer is usually below the ground.
Now that we have acquired a perspective on the distribution of mass in the atmosphere, we turn to an investigation of how the conservation of mass relates divergence/convergence and vertical motion.

### 6.4: Mass Continuity Equation

Imagine trying to fill a small basin with water from a hose. If there's a leak in the basin, then one needs to know both the inflow rate from the hose as well as the outflow rate through the leak in order to accurately gauge the filling rate. If the inflow rate is suddenly increased while the outflow rate remains the same, it’s simple to conclude that the mass of water in the basin will increase. If we designate the mass of water in the basin as $M_w$, then a simple expression of the mass continuity equation becomes

$$\frac{dM_w}{dt} = \text{Inflow Rate} - \text{Outflow Rate}$$
We can think of a slightly more abstract representation of this idea, illustrated in Figure 6.9, by considering an infinitesimal cube, fixed in space, through which air flows. The rate at which mass flows at the center of the cube (known as the mass flux) is given as the product of the velocity and the density of the air. If the inflow rate exceeds the outflow rate, we would say that mass is accumulating towards the center of the cube and thus there is net convergence into the infinitesimal cube. Conversely, if the outflow rate exceeds the inflow rate, we would say that mass is evacuating away from the center of the cube and thus there is net divergence out of the infinitesimal cube. In this way, we could define the divergence of the wind field as a measure of the rate at which mass is removed from a given volume of air and the convergence of the wind field as a measure of the rate at which mass is accumulates into a given volume of air. This physically suggests that the net rate of mass accumulation in the cube is represented by the divergence/convergence of the wind field. Writing this expression in terms of density gives

$$\frac{dp}{dt} \propto \text{Net Divergence}$$

In summary, the mass continuity equation states the regions of local convergence (divergence) leads to an increase (decrease) in mass. It is instructive at this point to consider the implications of the mass continuity equation for the atmosphere. A fluid in which individual parcels experience no change of density following the motion is known as an incompressible fluid. Even though the atmosphere is a compressible fluid, for many atmospheric phenomena the
compressibility is not a major physical consideration. In such cases, the mass continuity equation becomes a statement of zero velocity divergence throughout the depth of the entire atmosphere.

In synoptic meteorology, this statement can be explained in terms of **Dines compensation principle**. The Dines compensation principle states that there must be at least one level of nondivergence in the troposphere [typically called the **level of non-divergence** (LND)]. This level is usually around 550 mb, but can be highly variable depending on atmospheric stability. The compensation principle states that the convergence (divergence) that occurs above the LND tends to be offset by divergence (convergence) that occurs below the LND, as shown in Figure 6.10.

![Figure 6.10 Dines compensation principle applied to large-scale circulations](image)

Thus, if upper-level divergence occurs, the troposphere will attempt to compensate by initiating rising motion to “fill the void”. Increased convergence in the lower troposphere will usually result. This process is sometimes referred to as the **chimney effect**. When air is forced to rise vertically from the surface, it rises into regions where pressure is lower. If the divergence aloft is stronger than the convergence in the lower levels, surface pressure falls will occur since mass is being removed from the column by divergence.

If upper-level convergence occurs, the troposphere will attempt to compensate by initiating sinking motion (also called **subsidence**). Increased divergence in the lower troposphere will usually result. This process is sometimes referred to as the “damper effect”. If the convergence aloft is stronger than the divergence in the lower-levels, surface pressure rises will occur since mass is being added to the column by the convergence.
**6.4.1: Derivation of Mass Continuity Equation**

As before, consider an infinitesimal cube, fixed in space, through which air flows, as shown in Figure 6.9. For such a fixed control volume, the net rate of mass inflow through the sides must equal the rate of accumulation of mass within the volume. The rate of inflow of mass through the left-hand face per unit area is

\[
\left[ \rho u - \frac{\partial}{\partial x} (\rho u) \frac{\delta x}{2} \right]
\]

whereas the rate of outflow per unit area through the right-hand face is

\[
\left[ \rho u + \frac{\partial}{\partial x} (\rho u) \frac{\delta x}{2} \right]
\]

Because the area of each of these faces is \(\delta y \delta z\), the net flow rate into the volume due to the \(x\) velocity component is

\[
\left[ \rho u - \frac{\partial}{\partial x} (\rho u) \frac{\delta x}{2} \right] - \left[ \rho u + \frac{\partial}{\partial x} (\rho u) \frac{\delta x}{2} \right] = -\frac{\partial}{\partial x} (\rho u) \delta x \delta y \delta z
\]

Similar expressions hold for the \(y\) and \(z\) directions. Thus, the net rate of mass inflow is

\[
-\left[ \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) \right] \delta x \delta y \delta z
\]

and the mass inflow per unit volume is just \(-\nabla \cdot (\rho \mathbf{V})\), which must equal the rate of mass increase per unit volume. Now the increase of mass per unit volume is just the local density change \(\frac{\partial \rho}{\partial t}\). Therefore, we have

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0
\]

This is known as the **mass divergence form of the continuity equation**. An alternative form of the continuity equation is obtained by applying the vector identity

\[
\nabla \cdot (\rho \mathbf{V}) = \rho \nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla \rho
\]

This gives

\[
\frac{\partial \rho}{\partial t} + \rho \nabla \cdot \mathbf{V} + \mathbf{V} \cdot \nabla \rho = 0 \Rightarrow \frac{1}{\rho} D\rho = \nabla \cdot \mathbf{V}
\]
This is known as the **velocity divergence form of the continuity equation**. This states that the fractional rate of increase of the density *following the motion* of an air parcel is equal to velocity convergence.

Following the scaling analysis of section 6.1 and assuming that $|\rho' / \rho_0| \ll 1$, we can approximate the velocity divergence form of the mass continuity equation as

$$\frac{1}{\rho_0} \left( \frac{\partial \rho'}{\partial t} + \vec{V} \cdot \nabla \rho' \right) + \frac{w}{\rho_0} \frac{d \rho_0}{dz} + \nabla \cdot \vec{V} \approx 0$$

where $\rho'$ designates the local deviation of density from its horizontally averaged value, $\rho_0(z)$. For synoptic scale motions, $\rho'/\rho_0 \approx 10^{-2}$ so that using the characteristic scales in Section 5.1, we find that the first term has magnitude

$$\frac{1}{\rho_0} \left( \frac{\partial \rho'}{\partial t} + \vec{V} \cdot \nabla \rho' \right) \approx \frac{\rho' U}{\rho_0 L} \approx 10^{-7} \text{s}^{-1}$$

For motions in which the depth scale $H$ is comparable to the density scale height, $d \ln \rho_0 / dz \approx H^{-1}$, the second term scales as

$$\frac{w}{\rho_0} \left( \frac{d \rho_0}{dz} \right) \approx \frac{W}{H} \approx 10^{-6} \text{s}^{-1}$$

For synoptic scale motions, the third term scales as

$$\nabla \cdot \vec{V} \approx \frac{W}{H} + 10^{-1} \frac{U}{L} \approx 10^{-6} \text{s}^{-1}$$

Therefore, to a first approximation, the second and the third term balance in the continuity equation. To a good approximation then

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + w \frac{d}{dz} (\ln \rho_0) = 0 \Rightarrow \nabla \cdot (\rho_0 \vec{V}) = 0$$

Thus, for synoptic scale motions, the mass flux computed using the basic state density $\rho_0$ is nondivergent. This approximation is similar to the idealization of incompressibility. However, an *incompressible* fluid has constant density following the motion and thus, the velocity divergence will vanish, i.e. $\nabla \cdot \vec{V} = 0$. 

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6.5: The Geostrophic Approximation

Recall that the horizontal equations of motion can be written as

\[ \frac{d\vec{V}_H}{dt} = \vec{F}_{cor} + \vec{F}_{pressure} + \vec{F}_{friction} \]

If we perform a scaling analysis of this equation above the boundary layer (as shown in Section 6.1), we will see that the horizontal pressure gradient force is approximately balanced with the Coriolis force, leading to a condition known as geostrophic balance. Geostrophic balance can be expressed as

\[ f|\vec{V}| = -\nabla \Phi \]

The wind field associated with geostrophic balance is called the geostrophic wind.

\[ |\vec{V}_g| = \frac{\nabla \Phi}{f} \]

What kind of flow does geostrophic balance describe? We can get some insight into this question by considering the balance of forces involved. Consider the sea of sea-level isobars depicted in Figure 6.11. As mentioned earlier, the PGF vector is always directed from high to low pressure, perpendicular to the isobars as depicted in Figure 6.11. In order for there to be a force balance between the pressure gradient and Coriolis forces, the Coriolis force vector must be equal and opposite to the PGF vector as depicted. Since Figure 6.11 represents a hypothetical situation in the Northern Hemisphere, we know that the Coriolis force must be directed perpendicular to the motion of the air parcel and to the right. Consequently, as shown in Figure 6.11, the resulting geostrophic wind flows parallel to the isobars.

The same reasoning applies for upper-air charts where isohypses are plotted on isobaric maps. Here, the geostrophic wind is parallel to the geopotential height contours with a magnitude dependent on the magnitude of \( \nabla \Phi \). For synoptic-scale motions, the actual wind is close to the geostrophic wind (within 10-15% of the observed wind).
Given that geostrophy is a balance between the PGF and Coriolis forces, we might inquire under which conditions is geostrophic balance met. Since there is no mention of $d\vec{V}/dt$ in the geostrophic approximation, the geostrophic wind is only strictly valid in regions of zero wind acceleration. Since the wind is a vector quantity, with magnitude and direction, if either of those properties is changed over time, the wind has been accelerated. Thus, two broad categories of flow in the atmosphere will violate the geostrophic balance: those characterized by (1) wind speed changes along the flow, and/or (2) wind direction changes along the flow. The along-flow speed changes are most prominent in the vicinity of the local wind speed maxima known as jet streaks. Along-flow direction changes are most obvious in the vicinity of troughs and ridges in the pressure field. We will discuss the dynamics of these types of flows in a future chapter.
Suppose we examine the wind field within the boundary layer. In this region, it is expected that friction will be significant. Within the boundary layer, friction will slow down the winds, which will reduce the magnitude of the Coriolis force. Assuming that the pressure gradient force remains the same within the boundary layer, this implies that the magnitude of the pressure gradient force will exceed the magnitude of the Coriolis force. Therefore, the winds will no longer flow parallel to isobars. The winds will cross the isobars directed towards the lower pressure, as shown in Figure 6.12.

Due to the frictional turning of the wind such that it crosses the isobars, there will be rising motion near the surface low and sinking motion near the surface high. Thus frictional convergence produces rising motion. An example is winds blowing across Lake Michigan and into Michigan itself; friction is less over the lake than on land, so the air slows over Michigan and convergence between the lake and the land wind results. Likewise, frictional divergence produces subsidence. An example of frictional divergence is wind coming out of a mountainous area and onto flat terrain. Friction decreases over the flat area and the wind speeds up. The area between the slow mountain and the fast plains wind is an area of divergence.
6.6: Thermal Wind Balance

An analysis of the momentum equations for mid-latitude synoptic-scale motions demonstrate that the vertical equation of motion is dominated by two terms, the vertical pressure gradient force and the gravitational force, and the horizontal equations of motion are dominated by two terms, the horizontal pressure gradient force and the Coriolis force. The first condition leads to hydrostatic balance, whereas the second condition leads to a balanced condition called geostrophic balanced as discussed previously. Therefore, to a first order approximation, the mid-latitude atmosphere on Earth is in hydrostatic and geostrophic balance. Combining the horizontal balance condition of geostrophic balance and the vertical balance condition of hydrostatic balance leads to a single balance condition called thermal wind balance. Thermal wind balance indicates that there is a relationship between the vertical shear of the geostrophic wind and the horizontal temperature gradient. Thermal wind balance provides us with a powerful diagnostic tool for understanding the structure, dynamics, and evolution of mid-latitude weather systems. We will illustrate this important balance condition below.

Figure 6.13 Schematic introducing thermal wind balance

Recall that the thickness equation suggested that the thickness between two isobaric surfaces is smaller in a cold column of air than in a warm column. Consider a hypothetical example in which a cold column and a warm column are horizontally juxtaposed, as in Figure 6.13. The distance between the 1000 and 800 hPa surfaces must be larger in the warm air than...
the cold so that the 800 hPa surface slopes downward toward the cold air as illustrated. Similarly, the distance between the 800 and 500 hPa surfaces must be larger in the warm air than the cold and so the 500 hPa surfaces slopes even more dramatically downward toward the cold air. Thus, we find the slope of the isobaric surfaces increases with increasing height in the presence of a horizontal contrast in column average temperature. Of course, the slope of an isobaric surface is equivalent to the existence of a geopotential gradient along that surface since the geopotential is simply $g\Delta z$.

We now know that the pressure gradient force on an isobaric surface is related to the geopotential gradient on that surface. Thus, the increased slope to the isobaric surfaces in Figure 6.13 also means that the magnitude of the horizontal pressure gradient force increases with increasing height. Consequently, the geostrophic wind must be increasing with increasing height as well. Therefore, **there is a physical relationship between the vertical shear of the geostrophic wind (i.e. the manner in which the geostrophic wind changes with height) and the horizontal temperature gradient**. When the geopotential height gradient at one level of the atmosphere differs (in magnitude and/or direction) from that at another level, the geostrophic wind with at those levels will also be different. The **thermal wind** is defined as the vertical shear of the geostrophic wind (i.e. the change in the geostrophic wind with height).

We now explore the mathematical description of this relationship. From the discussion of geostrophic wind, we saw that the magnitude of the geostrophic wind speed was

$$|\vec{V}_g| = \frac{\|\nabla \Phi\|}{f} \approx \frac{1}{L} \frac{\|\delta \Phi\|}{f}$$

If we have geostrophic wind at two different isobaric levels, we can determine the magnitude of the thermal wind as $|\vec{V}_T| = |\vec{V}_{g,2} - \vec{V}_{g,1}|$ where

$$|\vec{V}_{g,1}| = \frac{1}{f} \frac{\delta \Phi_1}{L}, \quad |\vec{V}_{g,2}| = \frac{1}{f} \frac{\delta \Phi_2}{L}$$

(assuming that $\Phi_2 > \Phi_1$). Using the expression for the geostrophic wind, the magnitude of the thermal wind becomes

$$|\vec{V}_T| = \frac{1}{f} \frac{\delta \Phi_2 - \delta \Phi_1}{L}$$

where $\delta \Phi_2 - \delta \Phi_1$ is the difference in the geopotential gradient at two different isobaric levels, say 1000 and 500 mb. The change in geopotential height is the same as the change in thickness for the layer. Thus, as the thickness gradient increases, the thermal wind increases, which means
that the shear in the geostrophic wind increases. Using the expression for the thickness equation, we have

\[ |\vec{V}_T| = \frac{R_d}{f} \ln \left( \frac{P_1}{P_2} \right) \frac{\delta \overline{T}_{v,2} - \delta \overline{T}_{v,1}}{L} \]

where \( \overline{T}_v \) is the column average virtual temperature. Therefore, the thermal wind is proportional to the temperature gradient in the layer. To summarize, just as the geostrophic wind is proportional to the geopotential height gradient on an isobaric surface, the thermal wind is proportional to the thickness gradient between two isobaric surfaces, which is determined by the temperature gradient. The thermal wind relationship suggests that wind, geopotential height, and temperature are all locked together such that a change in one results in a change in the others.

Just as the geostrophic wind flows parallel to geopotential height contours, the thermal wind “flows” parallel to thickness contours, with colder layers (with lower thickness) to the left, and warmer layers (with greater thickness) to the right in the Northern Hemisphere. Similar to the Coriolis force and the geostrophic wind, the thermal wind is also reversed in the Southern Hemisphere. In the Northern Hemisphere, you can remember the simple rule that “with the thermal wind at your back, cold air is to your left!”

![Figure 6.14 Schematic maps of heights and thickness (contour interval = 20 m) and geostrophic and the thermal winds.](image)

In graphical form, the thermal wind vector is simply the vector difference between the geostrophic wind at some upper level in the atmosphere and the geostrophic wind at some lower level as shown in Figure 6.14. Mathematically, this is shown as \( |\vec{V}_T| = |\vec{V}_{g,2} - \vec{V}_{g,1}| \), where \( |\vec{V}_{g,2}| \)
is at a higher level than $|\vec{V}_{g,1}|$. Using vector addition and subtraction, we see that when thickness contours are parallel to geopotential height contours, the geostrophic winds at different levels will be parallel. This implies that the thermal wind is parallel to the geostrophic wind.

However, when thickness contours are not parallel to geopotential height contours, the wind changes direction with height, as shown in Figure 6.14. In Figure 6.14, we have a Northern Hemisphere case with 1000 mb geopotential heights oriented perpendicular to thickness contours for the mean 1000-500 mb layer. This has several consequences:

- When cold air is to the north, the top of the 1000-500 mb thickness layer tilts down to the north. This makes the 1000-500 mb contours perpendicular to those at 1000 mb.
- Adding the 1000-500 mb thickness (that slopes down to the north) to the 1000 mb heights (that slope down to the west) results in 500 mb heights that slope down to the northwest and a little more steeply than 1000 mb heights. This requires that the wind is somewhat stronger at 500 mb.
- The thermal wind is oriented parallel to the thickness contours of the 1000-500 mb layer, which are parallel to the isotherms for the average temperature in the layer.
- This case clearly demonstrates the simple rule that (in the Northern Hemisphere) ``with the thermal wind at your back, cold air is to your left!"

Once again, we see that the wind, geopotential height, and temperature fields are all dynamically linked. A change in one results in a change in the others. For this reason, the thermal wind equation is an extremely useful diagnostic tool, which is often used to check analyses of the observed wind and temperature fields for consistency.

![Figure 6.15 Relationship between turning of geostrophic wind and temperature advection: (a) backing of the wind with height and (b) veering of the wind with height](image-url)
Another important characteristic of the thermal wind is that it can tell us whether the average temperature in a layer is warming or cooling. Since the thermal wind always has cold air to its left in the Northern Hemisphere, and it is simply the difference between the geostrophic winds at two levels, we can look only at the winds at those levels and determine if the temperature advection into the region is warm or cold.

Because the thermal wind blows parallel to thickness contours with the warm air to the right facing downstream in the Northern Hemisphere, a geostrophic wind that turns counterclockwise with height (called backing) is associated with cold advection. In Figure 6.15, wind backed from north-northwest wind at $|\vec{V}_{g,1}|$ to a northwesterly wind at $|\vec{V}_{g,2}|$ where $|\vec{V}_{g,1}|$ is at a lower elevation than $|\vec{V}_{g,2}|$. In this scenario, the geostrophic wind at both levels flows from colder to warmer air, indicated cold advection. Conversely, a geostrophic wind that turns clockwise with height (called veering) is associated with warm advection.

It is therefore possible to obtain a reasonable estimate of the horizontal temperature advection and its vertical dependence at a given location solely from data on the vertical profile of the wind given by a single sounding. Alternatively, the geostrophic wind at any level can be estimated from the mean temperature field, provided that the geostrophic velocity is known at a single level. Thus, for example, if the geostrophic wind at 850 hPa is known and the mean horizontal temperature gradient in the layer 850-500 hPa is also known, the thermal wind equation can be applied to obtain the geostrophic wind at 500 hPa.
Because of the thermal wind relationship, meteorologists are often interested in how the wind direction changes with respect to time, or across a given space. The vertical variation of the wind field at a fixed location is sometimes plotted as a **hodograph**, as shown in Figure 6.16. A **hodograph** is the locus of points formed by the heads of all the wind vectors at a given location at adjacent heights. The difference between the wind vector aloft and the wind vector below, the vertical-shear vector, is tangent to the hodograph at any height. Thus, hodographs are useful not only because they depict the wind as a function of height, but they also because they show the vertical wind shear as a function of height. If the meteorological wind direction increases with time or height, then the wind field is veering with time or height. If the wind direction decreases with time or height, then the wind field is backing with time or height. For example, a southerly wind that later becomes easterly is thus said to have **backed**, and a vertical profile in which the surface wind is southeasterly and the wind aloft is westerly is thus said to **veer** with height. Thus, hodographs can be used to anticipate warm air advection and cold air advection into a region.

As we've shown above, the thermal wind relationship forms the cornerstone of modern dynamical meteorology as well as the first-order balance for the flow in the middle latitudes on Earth. This latter point is a direct consequence of the fact that the mid-latitude atmosphere is, to first order, geostrophically and hydrostatically balanced.
**6.6.1: Full Derivation of Thermal Wind Balance**

To derive the expression for thermal wind balance, it is convenient to change our vertical coordinates to height coordinates to isobaric coordinates. Writing the hydrostatic equation in isobaric coordinates gives

\[
\frac{\partial \Phi}{\partial P} = -\frac{R_d T_v}{P}
\]

The geostrophic wind can be determined from the expression of geostrophic balance in isobaric coordinates

\[
\vec{V}_g = \frac{\vec{k}}{f} \times \nabla_P \Phi
\]

The vertical derivative of the geostrophic wind relationship is

\[
\frac{\partial \vec{V}_g}{\partial P} = \frac{\vec{k}}{f} \times \nabla \frac{\partial \Phi}{\partial P}
\]

Using the isobaric form of the hydrostatic equation yields

\[
\frac{\partial \vec{V}_g}{\partial P} = -\frac{R_d \vec{k}}{f P} \times \nabla T_v
\]

Thus, we have the vertical shear of the geostrophic wind vector (i.e. the thermal wind). The component form of the thermal wind equation yields

\[
\frac{\partial u_g}{\partial P} = \frac{R_d}{f P} \frac{\partial T_v}{\partial y}
\]

\[
\frac{\partial v_g}{\partial P} = -\frac{R_d}{f P} \frac{\partial T_v}{\partial x}
\]

In Figure 6.13, we see that \( \partial T_v/\partial x > 0 \) and \( \partial v_g/\partial P < 0 \), which says that an increasing horizontal temperature gradient leads to stronger geostrophic wind aloft.

We can also express the thermal wind in terms of the geopotential difference using the thickness equation

\[
\frac{\partial u_g}{\partial P} = \frac{R}{f P} \frac{\partial T_v}{\partial y}
\]

\[
\frac{\partial v_g}{\partial P} = -\frac{R}{f P} \frac{\partial T_v}{\partial x}
\]

Integrating these expressions gives

\[
u_T = -\frac{R_d}{f P} \ln \left( \frac{P_0}{P_1} \right) = \frac{1}{f} \frac{\partial}{\partial y} (\Phi_1 - \Phi_0)
\]

\[
u_T = \frac{R_d}{f P} \ln \left( \frac{P_0}{P_1} \right) = \frac{1}{f} \frac{\partial}{\partial x} (\Phi_1 - \Phi_0)
\]
Thus, the vector form of the thermal wind relation is
\[ \vec{V}_T = \frac{R_d}{f} \ln \left( \frac{P_0}{P_1} \right) \hat{k} \times \nabla \vec{T} = \frac{\hat{k}}{f} \times \nabla (\Phi_1 - \Phi_0) \]

Thus, the thermal wind is proportional to the thickness gradient between two isobaric surfaces, which is determined by the temperature gradient, as suggested previously.
Chapter 7: Introduction to Quasigeostrophic Theory

The governing equations that describe the behavior of the atmosphere as described in Chapter 5 and 6 are quite complex. Even when written in Cartesian form and with simplifying assumptions such as the hydrostatic approximation, it remains difficult to conceptualize the dynamical essence of synoptic-scale weather systems using the full set of equations. However, early pioneers in meteorology, including Reginald Sutcliffe, Carl-Gustaf Rossby, Jule Charney, and Arnt Eliassen, recognized that the equations could be greatly simplified by utilizing the observation that the flow in synoptic-scale weather systems is approximately geostrophic. Jule Charney presented on the earliest quasigeostrophic (QG) derivations; the motive of this work was, in part, to provide a useful set of equations for early numerical weather prediction efforts.

By using a carefully designed set of assumptions, the governing equations can be simplified and combined in ways that retain the fundamental dynamics of weather systems and yet are simple enough to comprehend. The simplified QG framework provides many tools for dynamic analysis of the atmosphere while suggesting specific applications for weather forecasting. This system allows us to understand and diagnose the processes leading to vertical air motion and weather system development or decay, along with explaining why weather takes place.

Our aim is to understand the underlying assumptions of the QG equations and to understand the physical implications of QG theory. As we will see, the implications of QG theory form the conceptual framework of synoptic-scale forecasting and provide us physical insight into the nature of mid-latitude weather systems and the mid-latitude synoptic-scale flow. The QG framework allows us to simplify the equations of motion and reduce the system to two dependent variables that are useful in weather analysis and forecasting: the vertical velocity, which is closely related to the formation of clouds and precipitation, and the geopotential tendency, which is related to the development and movement of weather systems. We begin this chapter by examining departures from geostrophic balance in the atmosphere.
7.1: The Nature of the Ageostrophic Wind

As mentioned in Chapter 5, for small horizontal accelerations, we can approximate the horizontal, frictionless momentum equations with geostrophic balance. The magnitude of the geostrophic wind is given by

$$|V_g| = \frac{1}{f} |
\n\nHere, the geostrophic wind is parallel to the geopotential height contours with a magnitude dependent on the magnitude of $\n$. For synoptic-scale motions, the actual wind is close to the geostrophic wind (within 10-15\% of the observed wind). As mentioned in Chapter 5, the geostrophic wind is only strictly valid in regions of zero wind acceleration (i.e. constant velocity flow). The degree of departure from geostrophic balance that characterizes synoptic-scale motions can be assessed by considering the difference between the actual wind at a location and the calculated geostrophic wind at the same point. This difference is known as the ageostrophic wind $\n$, and is defined mathematically as $\n = \n - \n$. An important property of the geostrophic wind is that it is nondivergent. This is a consequence of the fact that the geostrophic flow is a non-accelerating flow. Since divergence implies a change in wind speed or wind direction, this requires that geostrophic wind is non-divergent. This is an extremely important property of the geostrophic wind because it demonstrates that any divergence in the synoptic-scale wind field is due to the presence of ageostrophic motion. Since vertical motion is caused by convergence/divergence in the wind field according to the Dines compensation principle, then the ageostrophic wind determines the distribution of vertical velocity in the atmosphere. Thus, it is the ageostrophic wind that is entirely responsible for the distribution of cyclones, anticyclones, clouds, and precipitation in the atmosphere. The ramification of this statement is profound. Despite the fact that the mid-latitude atmosphere is predominately in geostrophic balance, all of the important weather with which we are confronted develops as a direct result of the often relatively small ageostrophic portion of the wind.

The discussion above indicates that the magnitude of the ageostrophic wind is related to the magnitude of the acceleration vector. Using vector form of the horizontal momentum equations and the geostrophic wind equations, it can be shown that the ageostrophic wind has the following properties:

- The ageostrophic wind is a measure of the horizontal acceleration in the atmosphere
- The ageostrophic wind is perpendicular to the horizontal acceleration vector
• The ageostrophic wind is directed to the left of the acceleration vector in the Northern Hemisphere.

The last point is very important because it describes how the wind field responds to changes in the pressure field. If wind flow is from an area of small pressure gradient to tight pressure gradient, the wind must accelerate in order to approach geostrophic balance. The ageostrophic wind accomplishes this by blowing to the left (toward lower pressure). Conversely, if wind flow is from an area of tight pressure gradient to an area of small pressure gradient, the wind must decelerate to approach geostrophic balance. The ageostrophic wind will blow toward higher pressure (and still to the left of the acceleration). To examine the nature of the ageostrophic flow, we will examine two broad classes of circumstances in which geostrophic balance is violated: the presence of curvature in the flow and the presence of along-flow speed change.

**7.1.1: Derivation of Ageostrophic Wind**

Since our analysis of the wind field is usually done on upper level isobaric maps, it will be convenient to write the horizontal momentum equations in isobaric coordinates (i.e. the vertical coordinate will be in terms of pressure). In order to recast the momentum equations in isobaric coordinates, we must convert the pressure gradient force term into an equivalent expression in isobaric coordinates. This is done by considering the differential $dp$ on a constant pressure surface:

$$ dp = \left( \frac{\partial p}{\partial x} \right)_{y,z} \, dx_p + \left( \frac{\partial p}{\partial y} \right)_{x,z} \, dy_p + \left( \frac{\partial p}{\partial z} \right)_{x,y} \, dz_p $$

Since there’s no change in pressure on an isobaric surface, then $dp = 0$ so that

$$ 0 = \left( \frac{\partial p}{\partial x} \right)_{y,z} \, dx_p + \left( \frac{\partial p}{\partial y} \right)_{x,z} \, dy_p + \left( \frac{\partial p}{\partial z} \right)_{x,y} \, dz_p $$

Next, we expand $dz_p$ as a function of $x$ and $y$ to yield

$$ 0 = \left( \frac{\partial p}{\partial x} \right)_{y,z} \, dx_p + \left( \frac{\partial p}{\partial y} \right)_{x,z} \, dy_p + \left( \frac{\partial p}{\partial z} \right)_{x,y} \left[ \left( \frac{\partial z}{\partial x} \right)_{y,p} \, dx_p + \left( \frac{\partial z}{\partial y} \right)_{x,p} \, dy_p \right] $$

This can be rearranged into

$$ 0 = \left[ \left( \frac{\partial p}{\partial x} \right)_{y,z} + \left( \frac{\partial p}{\partial z} \right)_{x,y} \left( \frac{\partial z}{\partial x} \right)_{y,p} \right] \, dx_p + \left[ \left( \frac{\partial p}{\partial y} \right)_{x,z} + \left( \frac{\partial p}{\partial z} \right)_{x,y} \left( \frac{\partial z}{\partial y} \right)_{x,p} \right] \, dy_p $$
Since this statement is true for all \( dx \) and \( dy \), this implies that

\[
\left( \frac{\partial p}{\partial x} \right)_{y,z} = - \left( \frac{\partial p}{\partial z} \right)_{x,y} \left( \frac{\partial z}{\partial x} \right)_{y,p} \quad \text{and} \quad \left( \frac{\partial p}{\partial y} \right)_{x,z} = - \left( \frac{\partial p}{\partial z} \right)_{x,y} \left( \frac{\partial z}{\partial y} \right)_{x,p}
\]

Using the hydrostatic equation and dividing by \( \rho \) gives

\[
- \frac{1}{\rho} \frac{\partial p}{\partial x} = - g \left( \frac{\partial z}{\partial x} \right)_{y,p} = - \frac{\partial \Phi}{\partial x}_{y,p}
\]

\[
- \frac{1}{\rho} \frac{\partial p}{\partial y} = - g \left( \frac{\partial z}{\partial y} \right)_{x,p} = - \frac{\partial \Phi}{\partial y}_{y,p}
\]

The momentum equations in isobaric coordinates can be written as

\[
\frac{Du}{Dt} = - \frac{\partial \Phi}{\partial x} + f v + v \nabla^2 u
\]

\[
\frac{Dv}{Dt} = - \frac{\partial \Phi}{\partial y} - f u + v \nabla^2 u
\]

For small accelerations, the condition for geostrophic balance in isobaric coordinates is given by

\[
f v \approx \frac{\partial \Phi}{\partial x}, \quad f u \approx - \frac{\partial \Phi}{\partial y}
\]

In vector form, the geostrophic wind in isobaric coordinates can be written as

\[
\vec{V}_g = \frac{\vec{k}}{f} \times \nabla \rho \Phi
\]

We can also derive the mass continuity equation in isobaric coordinates. Consider an infinitesimal cube, fixed in space, through which air flows as in Figure 7.1
Using the hydrostatic equation, we can write the control volume as

$$\delta V = \delta x \delta y \delta z = -\frac{\delta x \delta y \delta p}{\rho g}$$

The Lagrangian rate of change of mass (per unit mass) is given by

$$\frac{1}{\delta M} \frac{D(\delta M)}{Dt} = -\frac{g}{\delta x \delta y \delta p} \frac{D}{Dt} \left( -\frac{\delta x \delta y \delta p}{g} \right)$$

Applying the chain rule gives

$$\frac{1}{\delta x \delta y \delta p} \left[ \frac{D(\delta x)}{Dt} \delta y \delta p + \frac{D(\delta y)}{Dt} \delta x \delta p + \frac{D(\delta p)}{Dt} \delta x \delta y \right] = 0$$

This can be simplified to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

where $\omega \equiv Dp/Dt$ (usually called the “omega” vertical motion) is the pressure change following the motion, which plays the same role in the isobaric coordinate system that $w = Dz/Dt$ plays in height coordinates. Writing the above expression in vector form gives the mass continuity equation

$$\nabla \cdot \vec{V}_h = -\frac{\partial \omega}{\partial p}$$
Substituting $\vec{V}_h = \vec{V}_g + \vec{V}_{ag}$ gives

$$\nabla \cdot \vec{V}_h = \nabla \cdot (\vec{V}_g + \vec{V}_{ag}) = -\frac{\partial \omega}{\partial p}$$

Now

$$\nabla \cdot \vec{V}_g = \frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = -\frac{1}{f} \frac{\partial^2 \Phi}{\partial x \partial y} + \frac{1}{f} \frac{\partial^2 \Phi}{\partial y \partial x} = 0$$

Since the geostrophic wind is nondivergent, we have

$$\nabla \cdot \vec{V}_{ag} = -\frac{\partial \omega}{\partial p}$$

We are now in the position to derive the expression for the ageostrophic wind. We begin with the frictionless equations of motion

$$\frac{d\vec{V}}{dt} = -\vec{V}_p \Phi - f \hat{k} \times \vec{V}$$

Taking the vertical cross product of the horizontal momentum equation gives

$$\frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = -\frac{\hat{k}}{f} \times \vec{V}_p \Phi - \frac{\hat{k}}{f} \times f \hat{k} \times \vec{V} = -\frac{\hat{k}}{f} \times \vec{V}_p \Phi + \vec{V}$$

Using the definition of the geostrophic wind, we have

$$\frac{\hat{k}}{f} \times \frac{d\vec{V}}{dt} = \vec{V} - \vec{V}_g = \vec{V}_{ag}$$

Based on the properties of the cross product, we see that
- The ageostrophic wind is a measure of the horizontal acceleration in the atmosphere
- The ageostrophic wind is perpendicular to the horizontal acceleration vector
- The ageostrophic wind is directed to the left of the acceleration vector in the Northern Hemisphere.
7.1.2: Curvature in the Flow

Consider the flow through an upper-tropospheric trough-ridge couplet where the wind speed is constant and everywhere parallel to the geopotential height lines as shown in Figure 7.2. Under such circumstances, the acceleration of the wind will be entirely a consequence of directional changes. Thus, between points A and B in Figure 7.2, a southwestward-directed acceleration is required to turn the wind from westerly at point A to northwesterly at point B. There is no direction change between points B and C and, thus, no acceleration vector. A northeastward-directed acceleration is required to turn the northwesterly wind at point C to a westerly direction at point D. In order to turn the westerly at point D to a southwesterly at point E, a northwestward-directed acceleration is required. No change in direction exists between points E and F but a change from southwesterly at F to westerly at point G requires a southeastward-directed acceleration as shown.

Given the four acceleration vectors drawn in Figure 7.2, it is simple to draw the ageostrophic winds in this trough–ridge couplet. The ageostrophic winds clearly converge on the western side of the upper trough (on its upstream side) leading to downward vertical motion in the column in that location (according to the Dines compensation principle). Meanwhile, the divergence of the ageostrophic winds on the downstream side of the upper trough is associated with upward vertical motions in the column in that location. This result provides a first insight into the physical reason why inclement weather is often found downstream of upper-level trough axes while clear skies are often found downstream of upper-level ridge axes. This basic
relationship lies at the heart of understanding the distribution of sensible weather in the middle latitudes.

### 7.1.3: Speed Changes Along the Flow

![Figure 7.3](image-url)

Figure 7.3 The 300 hPa isotachs (solid lines) and wind vectors associated with a straight jet at 0000 UTC 12 November 2003. Isotachs are labeled in m/s and contoured every 10 m/s starting at 50 m/s. Only wind vectors greater than 40 m/s are shown. Thick black arrows indicate the direction of the acceleration vector at the entrance region (solid black circle) and exit regions (open circle) of the jet. The gray shaded arrow is the resultant ageostrophic wind vector at both conditions. C and D represent the locations of 300 hPa ageostrophic convergence and divergence, respectively.

Consider the flow through an isolated jet streak, which is defined as a local maximum within the jet streak. Figure 7.3 is the isotach distribution associated with an isolated wind speed maximum at 300 hPa in the northern hemisphere. The dashed line drawn perpendicular to the jet axis divides the jet into the so-called **entrance region** to its left and the **exit region** to its right. A parcel of air located on the western edge of the entrance region (indicated by the solid circle in Figure 7.3) would quite experience an acceleration in the direction of the flow at that location. Hence, the vector $d\vec{V}/dt$ points eastward toward the center of the jet streak. Consequently, the ageostrophic wind vector $\vec{V}_{ag}$ points northward at the indicated point. The result of this distribution of ageostrophic winds in the entrance region of the jet is that there is convergence of air at 300 hPa to the north of the indicated position and divergence of air at 300 hPa to the south of the indicated position. Given that 300 hPa is nearly at the top of the troposphere, upper-level...
divergence (convergence) is associated with upward (downward) vertical motion in the intervening column.

A parcel of air located on the eastern edge of the exit region (indicated by the open circle in Figure 7.3) would quite obviously experience a deceleration in the direction opposite the flow at that location. Hence, the vector $d\vec{V}/dt$ points westward toward the center of the jet streak. Consequently, the ageostrophic wind vector $\vec{v}_{ag}$ points southward at the indicated point. The result of this distribution of ageostrophic winds in the exit region of the jet is that there is convergence of air at 300 hPa to the south of the indicated position and divergence of air at 300 hPa to the north of the indicated position. Upward vertical motion occurs in the column beneath the upper divergence maxima. This result leads to the observed ageostrophic circulation in jet streaks and the straight jet streak model, which we will discuss in more detail in a later chapter.

### 7.1.4: Ageostrophic Wind and Pressure Tendency

Recall that the ageostrophic wind is related to the horizontal acceleration vector. The famous British meteorologist R. C. Sutcliffe reasoned that surface pressure falls resulted from vertical differences in mass divergence in a column. Larger mass divergence aloft than at the surface resulted in surface pressure falls and vice versa for surface pressure rises. Such differences in divergence could be related to differential accelerations at the surface and aloft through the ageostrophic wind. Thus, Sutcliffe argued that isolation of the acceleration vector could give insights into the sense of the vertical motion in an atmospheric column. Therefore, this suggests that the ageostrophic wind is connected to surface pressure tendency (and geopotential height tendency). In particular, Sutcliffe demonstrated that the divergence (convergence) of the ageostrophic wind is associated with surface pressure and geopotential height falls (rises).

We can apply Sutcliffe's insight to Figures 7.2 and 7.3. For Figure 7.2, Sutcliffe's insight suggests that there will be geopotential height falls (rises) downstream (upstream) of the upper-level trough. Similarly, for Figure 7.3, Sutcliffe's insight suggest that there will geopotential height falls in the right-entrance and left-exit regions of the jet streak, while right-exit and left-entrance regions are associated with geopotential height rises.
7.2: Vorticity and Divergence

Another central aspect of synoptic-scale midlatitude motions is the relationship between vorticity and divergence. This can be demonstrated by examining the rate of change of relative vorticity. It can be shown that the rate of change of relative vorticity is given by the sum of three processes: (1) advection, (2) divergence, and (3) tilting term. Let’s examine both advection terms have a familiar physical interpretation as they both describe advective processes, so we will examine the divergence term and the tilting term.

The effect of vorticity advection on the evolution of vorticity can be illustrated for midlatitude systems as shown in Figure 7.4. In the region upstream of the 500-hPa trough, the geostrophic wind is directed from the relative vorticity minimum at the crest of the ridge toward the relative vorticity maximum at the base of the trough, which produces anticyclonic vorticity advection (or negative vorticity advection). Hence, upstream of the trough, vorticity advection tends to decrease the local vorticity. Similar arguments (but with reversed signs) apply to region downstream of the trough. Therefore, vorticity advection tends to move the vorticity pattern and hence the troughs and ridges eastward (downstream).
Figure 7.5 (a) Illustration of the effect of divergence (D) on vorticity. Original fluid ring is lightly shaded and bordered by the large circulation arrows. Fluid ring at some later time is darkly shaded and bordered by smaller circulation arrows. (b) As for (a) but for condition of horizontal convergence (C)

Figure 7.5 illustrates the effects of divergence on the change of vorticity within a fluid. As illustrated in Figure 7.5(a), when divergence occurs in a fluid, then the area enclosed by a fluid ring increases with time. Consequently, the vorticity becomes more anticyclonic with time. The opposite is true for a fluid characterized by convergence as illustrated in Figure 7.5(b). Horizontal convergence will produce a cyclonic tendency in the vorticity consistent with the shrinking of the area enclosed by the fluid element and consequent increase in the ratio of circulation to area. The effect of the divergence term is the fluid analog of a figure skater whose rate of rotation increases (decreases) when she pulls (extends) her arms to (from) her side, decreasing (increasing) the radius of rotation. Since no torques are applied to the skater, the smaller (larger) radius of rotation requires a larger (smaller) angular velocity. So, in the end, we can say that: convergence (divergence) spins up cyclonic (anticyclonic) vorticity. There are profound implications to these statements with regard to mid-latitude weather systems. Surface low-pressure centers are characterized by convergence and thus tend to be foci for the production of low-level cyclonic vorticity, as shown in Figure 7.6. Just the opposite is true for surface anticyclones.
Figure 7.6 Schematic of the relationship between vorticity advection, divergence, and vertical motion

Tilting describes the effects of vertical shear on the change in vorticity. This term describes how horizontal vorticity can be projected to the vertical axis to produce vertical vorticity. This term is very important for mesoscale phenomena, such as thunderstorms, but this term tends to be small for midlatitude synoptic-scale motions. For this reason, we will not go into depth with the qualitative description of this term.

In comparing the relative magnitude of each effect, a scaling analysis shows that the dominant processes which change relative vorticity are divergence and horizontal advection. Therefore, the Lagrangian rate of change of the relative vorticity following the horizontal motion on the synoptic scale is largely a consequence of the generation or destruction of vorticity through horizontal divergence. Horizontal vorticity advection governs the distribution and location of relative vorticity. Another way to state this is that the local rate of change of vorticity is given by the sum of the advection of the vorticity plus the concentration (or dilution) by stretching or shrinking of fluid columns (i.e., the divergence effect).

The presence and centrality of horizontal divergence in the generation of vorticity suggests that an important set of relationships exists in fluids. The rotation of a fluid depends upon the presence of divergence in that fluid. The presence of divergence in that fluid requires, by continuity, that the fluid also possesses regions of upward and downward motions. In the atmospheric fluid, those upward and downward motions, and the associated adiabatic warming and cooling that goes along with them, are associated with phase changes of the water substance and the delivery of our sensible weather.
7.2.1: Derivation of the Vorticity Equation

To derive the vorticity equation, we start with the momentum equations in isobaric coordinates:

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \omega \frac{\partial u}{\partial p} - f v &= - \frac{\partial \phi}{\partial x} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \omega \frac{\partial v}{\partial p} + f u &= - \frac{\partial \phi}{\partial y}
\end{align*}
\]

Taking the \( y \)-derivative of the zonal equation and the \( x \)-derivative of the meridional equation gives:

\[
\begin{align*}
\frac{\partial}{\partial t} \left( \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left( u \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( v \frac{\partial u}{\partial y} \right) + \frac{\partial}{\partial y} \left( \omega \frac{\partial u}{\partial p} \right) - f \frac{\partial v}{\partial y} &= - \frac{\partial^2 \phi}{\partial x \partial y} \\
\frac{\partial}{\partial t} \left( \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial x} \left( u \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial x} \left( v \frac{\partial v}{\partial y} \right) + \frac{\partial}{\partial x} \left( \omega \frac{\partial v}{\partial p} \right) + f \frac{\partial u}{\partial x} &= - \frac{\partial^2 \phi}{\partial x \partial y}
\end{align*}
\]

Subtracting the equations and simplifying gives:

\[
\frac{\partial \zeta}{\partial t} = -u \frac{\partial \zeta}{\partial x} - v \frac{\partial \zeta}{\partial y} - \omega \frac{\partial \zeta}{\partial p} - (f + \zeta) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) + \hat{k} \cdot \left( \frac{\partial \vec{V}}{\partial p} \times \nabla \omega \right)
\]

This is known as the vorticity equation. The first two terms on the right hand side of the vorticity equation correspond to the horizontal advection of vorticity, as discussed previously. The third term on the right hand side corresponds to the advection of vorticity by the vertical wind. The fourth term describes the generation of vorticity by divergence, which we call the divergence term. The fifth term on the right hand side describes the generation of vertical vorticity by the vertical shear, which we call the tilting term.

In Chapter 6, the equations of motion were simplified for synoptic-scale motions by evaluating the order of magnitude of various terms. The same technique can also be applied to the vorticity equation. Characteristic scales for the field variables based on typical observed magnitudes for synoptic-scale motions are chosen as follows:
Characteristic horizontal velocity

Characteristic omega velocity

Characteristic length scale of synoptic-scale features

Characteristic depth scale

Characteristic horizontal pressure fluctuation

Characteristic pressure

Characteristic time scale

Coriolis parameter

Using these scales to evaluate the magnitude of the terms in the vorticity equation, we first note

\[ \zeta \sim \frac{U}{L} \approx 10^{-5} s^{-1} \Rightarrow \frac{\zeta}{f_0} \approx \frac{U}{f_0 L} \approx 10^{-1} \]

Therefore, for midlatitude synoptic-scale systems, the relative vorticity is small compared to the planetary vorticity. For such systems, \( \zeta \) may be neglected compared to \( f \) in the divergence term in the vorticity equation.

\[ (\zeta + f) \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \]

The magnitude of the various terms in the vorticity equation can now be estimated as

\[ \frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} \approx \frac{U^2}{L^2} \approx 10^{-10} s^{-2} \]

\[ \omega \frac{\partial \zeta}{\partial p} \approx \frac{W U}{P_0 L} \approx 10^{-11} \]

\[ f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \approx \frac{f_0 U}{L} \approx 10^{-9} s^{-2} \]

\[ \left( \frac{\partial \omega}{\partial x} \frac{\partial v}{\partial z} - \frac{\partial \omega}{\partial y} \frac{\partial u}{\partial z} \right) \approx \frac{W U}{P_0 L} \approx 10^{-11} s^{-2} \]

The scale analysis of the vorticity equation indicates that synoptic-scale motions are approximately non-divergent. Retaining only the terms greater than \( 10^{-10} s^{-2} \) in the vorticity equation yields the approximate form valid for synoptic scale motions.
\[
\frac{D_h\zeta}{Dt} = -f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)
\]

where \( D_h/Dt = \partial/\partial t + u \partial/\partial x + v \partial/\partial y \). This expression states that the change in vorticity following the horizontal motion on the synoptic scale is given approximately by the concentration or dilution of planetary vorticity caused by the convergence or divergence of the horizontal flow.

### 7.3: The Diagnosis of Synoptic-Scale Vertical Motion

The most important application to quasigeostrophic theory is the diagnosis of vertical motion. The vertical velocity field is important because it is responsible for the amplification and/or de-amplification of middle tropospheric troughs and ridges. As a result, if we want to know something about how the amplitude of the mid-latitude trough/ridge pattern is evolving, we need to know how the vertical velocity varies with respect to pressure. Moreover, synoptic-scale vertical motion plays a role in the formation and evolution of clouds and precipitation. The dynamics of quasigeostrophic theory leads to two important diagnostic equations called the **QG omega equation** and the **QG height tendency equation**. These equations demonstrate that vertical motion and height tendency are dynamically caused by two mechanisms: vorticity advection and thermal advection. We will give a qualitative description of these forcing terms below.

### 7.3.1: Vorticity Advection

![Figure 7.7 Schematic 500-hPa geopotential field showing an upper-level trough and ridge couplet.](image)
One way of examining how vorticity advection is connected to vertical motion is to consider a trough embedded in the westerlies, as in Figure 7.7. As mentioned in previous chapters, the base of the trough contains strong curvature vorticity and thus, the trough can be viewed as a region of enhanced vorticity. Now consider a parcel entering the trough from behind. This parcel has some value of vorticity as it enters the trough. Along its trajectory, it is encountering more and more cyclonic vorticity values. If it is to stay in equilibrium with its environment, it must increase its vorticity. How can it do so? According to our discussion of the vorticity equation, a fluid gains vorticity through horizontal convergence. Using the analogy of the ice skater given above, the air parcel converges to stay in equilibrium with its environment and conservation of angular momentum requires the rotation associated with the parcel to increase. By this reasoning, parcels entering the backside of the trough, in a region of anticyclonic vorticity advection (also called negative vorticity advection (NVA) in the Northern Hemisphere) (that is, vorticity values are becoming more cyclonic along the flow) have to converge. By the same argument, in the region of cyclonic vorticity advection (also called positive vorticity advection (PVA) in the Northern Hemisphere) on the downstream side of the trough, parcels are diverging. In this way, PVA implies divergence and NVA implies convergence at any level in the atmosphere.

The corresponding vertical motion due to vorticity advection depends upon the Dines compensation principle. If PVA (NVA) occurs above the level of non-divergence, then PVA (NVA) will be associated with rising (sinking) motion. The same logic implies that if significant PVA (NVA) occurs below the level of non-divergence, then PVA (NVA) will be associated with sinking (rising) motion. Generally speaking, PVA at 500 mb is typically associated with PVA increasing with height. Most of this is due to the increase of westerly wind speed with height and hence, it is only applicable in mid-latitudes. Therefore, the common assumption is that CVA is associated with QG forcing for upward motion. For this reason, the vorticity advection term is also called the upper level divergence term.

An operational forecaster recognizes PVA by locating:

- The downstream region of relatively higher vorticity values along with the gradient of vorticity values,
- The wind speed of air through the gradient of vorticity, and
- The angle the airflow goes through the vorticity gradient

PVA generally occurs in the downstream region (region where airflow is moving away from highest values of vorticity) of a vorticity maximum. PVA is maximized by the combination of:

- high values of vorticity, with more importantly a large rate of change (gradient) of vorticity,
- a strong, downwind of vorticity max, airflow through the high gradient of vorticity,
- an airflow perpendicular to the vorticity gradient

Vorticity advection also has an effect on geopotential height tendency within the atmosphere. Since PVA is associated with divergence, then PVA must also be associated with geopotential height falls due to the relationship between ageostrophic wind and pressure tendency. If PVA increases with height (which is in accordance with westerly wind increasing with height), this means that geopotential heights are falling more with height, so that the depth of the atmospheric column between the surface and some upper level is decreasing. According to the thickness equation, the thickness of a layer between two pressure levels is related to the mean temperature in between the pressure levels. If the thickness decreases, the mean temperature is also decreasing. In an adiabatic atmosphere, in the absence of thermal advection, the column must cool by producing rising motion. The same argument can be used to indicate that NVA is associated with geopotential height rises.

These explanations also imply that the advection of vorticity above the level of non-divergence will result in a response at the surface which will attempt to offset the effects of the advection. More specifically, PVA is indicating if rising motion/falling pressure at the surface. Thus, PVA tends to enhance a surface low by making it spin faster.

**7.3.2: Thermal Advection**

In many situations, thermal advection (or thickness advection), especially associated with low-level flow, is actually the dominant physical effect associated with large-scale ascent. The contribution associated with thermal advection typically (not always!) decreases with height, as the normal situation is for the atmosphere to become more barotropic with height (i.e. isotherms and lines of constant geopotential heights tend to be parallel at upper levels in the atmosphere). However, this may be counteracted, to some extent, by the typical increase of westerly wind speeds with height.

How, physically, does thermal advection contribute to vertical motion? In QG theory, the flow is assumed to be adiabatic, meaning that a parcel's potential temperature does not change. For adiabatic flow, warm advection on an isobaric surface implies ascent, while cold advection implies descent. In fact, the adiabatic assumption often is a good one, even on scales smaller than implied by the full set of QG assumptions. Hence, QG implied vertical motion may be reasonable on scales where the QG assumptions are not met, largely because of the importance of the thermal advection term and its connection to adiabatic flow.
As indicated in Figure 7.8, rising motion will occur to the east of the surface low in advance of the warm front, where the strongest positive thickness advection exists. Conversely, sinking motion will occur to the west of the surface low in the region of strongest cold advection. Because warm advection under the 500-hPa ridge (to the east of the surface low) will result in an increase in the 500-1000-hPa thickness field (via the thickness equation), the height of the 500-hPa surface will bow upward (increase) at this location. This leads to larger anticyclonic vorticity at the crest of the ridge. In the absence of NVA at the ridge axis, the only means of decreasing the relative vorticity is via horizontal divergence. Continuity requires upward motion beneath the upper-level divergence (500-hPa ridge axis) and corresponding low-level convergence. Therefore, low-level warm advection (or positive thickness advection) is associated with rising motion and geopotential height rises above the level of maximum thickness.

Because cold air advection under the 500-hPa trough (to the west of the surface low) will result in a decrease in the 500-1000-hPa thickness field (via the thickness equation), the height of the 500-hPa surface will bow downward (decrease) at this location, thus amplifying the mid-level trough. As the mid-level trough amplifies, this leads to larger cyclonic vorticity at the base of the trough. In the absence of PVA at the trough axis, the only means of increasing the relative vorticity is via horizontal convergence. Continuity requires downward motion beneath the upper-level convergence (500-hPa trough axis) and corresponding low-level divergence. Therefore, low-level cold advection (or negative thickness advection) is associated with sinking motion and geopotential height falls below the level of maximum advection.
It is important to realize that the vertical motions induced by the low-level thermal forcing acts to create adiabatic temperature changes that offset the initial temperature perturbations. More specifically, the upward motion (adiabatic cooling) at the 500-hPa ridge axis counteracts the warm advection. In addition, sinking air (adiabatic warming) at the 500-hPa trough axis counteracts the low-level cold advection. In essence, the ageostrophic motion is trying to return the atmosphere to hydrostatic balance after the initial thermal perturbations. These considerations suggest that thermal advection intensifies upper-level troughs and ridges in building synoptic systems.

As mentioned above, the effects of thermal advection and vorticity advection can counteract each other. Therefore, it becomes necessary to quantify the effects of thermal advection and vorticity advection in producing vertical motion. In operational meteorology, this is done by examining the so-called $\mathbf{Q}$-vectors.

### 7.3.3: $\mathbf{Q}$ vectors

![Diagram showing $\mathbf{Q}$ vectors](image)

Figure 7.9 Jet entrance region in the northern hemisphere. Thick solid lines are 500 hPa geopotential height, dashed lines are 1000-500 hPa thickness, and thin solid lines are isotachs of the geostrophic wind. Point C is mentioned in the explanation given in the notes.
Consider the jet entrance region depicted in Figure 7.9. The confluent, geostrophic wind field depicted there acts to tighten the horizontal temperature gradient at C. Any such increase in the magnitude of the temperature gradient forces an increase in the geostrophic vertical shear via the thermal wind relationship. Simultaneously, the geostrophic wind advects lower geostrophic momentum (quantified by the isotachs of the geostrophic wind) into the jet core. The momentum advection tends to decrease the wind speed at C and, thus, contributes to a decrease in the vertical shear of the geostrophic wind in that column. Thus, the very same geostrophic flow that serves to increase the magnitude of the horizontal temperature gradient at C also serves to decrease the vertical shear of the geostrophic wind at C via negative geostrophic momentum advection. This set of circumstances presents a paradox: that is, on the one hand, geostrophic temperature advection should increase the thermal wind at C and, on the other, geostrophic momentum advection should decrease it at C. So, the geostrophic wind actually destroys thermal wind balance by affecting opposite signed changes to the two components of that balance. Since the thermal wind balance is a form of the geostrophic balance, it can therefore be said that the geostrophic wind destroys itself! We will refer to this property of the geostrophic flow as the geostrophic paradox.

Interestingly, however, observations suggest that the synoptic-scale flow in the middle latitudes is very nearly in geostrophic balance at all times. How can this be in the face of what we have just described? There must be another portion of the flow that acts to maintain the geostrophic balance in the face of its self-destructive tendency. That portion of the flow is the forced, ageostrophic, secondary circulation. Since the geostrophic flow tends to create thermal wind imbalance, the forced secondary circulation must bring the flow back toward a state of geostrophic balance. This may be accomplished if the secondary circulation counteracts the tendencies induced by the geostrophic wind itself. Therefore, the secondary, ageostrophic circulation operating in the vicinity of the jet entrance region depicted in Figure 7.9 must simultaneously (1) decrease the magnitude of the horizontal temperature gradient, and (2) increase the vertical shear. To quantify the self-destructive geostrophic tendency (and thus to quantify the geostrophic paradox), operational meteorologists use a derived measure of vertical motions called $\vec{Q}$-vectors.

$\vec{Q}$-vectors highlight areas where the thermal wind balance is being disrupted, which will cause compensating force, ageostrophic, secondary circulations. Thus $\vec{Q}$ represents the degree of thermal wind imbalance caused by the geostrophic wind. The $\vec{Q}$-vector approach combines the two key contributions to vertical motion: temperature advection and convergence/divergence due to changes in vorticity advection with height. $\vec{Q}$-vectors are interpreted as follows: $\vec{Q}$-vector convergence (divergence) indicates rising (sinking) motion at that level.
Figure 7.10 (a) $\vec{Q}$-vectors for the confluent jet entrance region depicted in Figure 7.9. Vertical cross-section along line $A-B$ is shown in (b). (b) Vertical cross-section along line $A-B$ in (a). Black arrows represent the vertical and horizontal branches of the secondary, ageostrophic circulation associated with the $\vec{Q}$-vector distribution in (a). Gray arrows represent the direction of the horizontal branch of the forced circulation before the Coriolis force turns it to the right. See text for explanation.

Figure 7.10(a) shows the $\vec{Q}$-vectors for the confluent jet entrance of Figure 7.9. This configuration of $\vec{Q}$-vectors results in $\vec{Q}$ convergence in the warm air and $\vec{Q}$ divergence in the cold air. Consequently, we have diagnosed a thermally direct, secondary, vertical circulation in which the warm air rises and the cold air sinks (Figure 7.10b). Such a secondary ageostrophic
circulation achieves two important modifications of the environment. First, adiabatic cooling of the rising warm air and adiabatic warming of the sinking cold air decrease the magnitude of $\nabla T$. This exactly counteracts the tendency of the geostrophic temperature advection in the confluent flow. Second, under the influence of the Coriolis force, the horizontal branches of this secondary ageostrophic circulation tend to increase the vertical wind shear — exactly counteracting the tendency of the geostrophic momentum advection in the confluent flow! Thus, the secondary ageostrophic circulation diagnosed with the $\vec{Q}$-vectors is precisely that necessary to restore the thermal wind balance in the face of the geostrophic wind’s tendency to destroy the balance.

As mentioned above, the $\vec{Q}$-vector approach is good because vorticity advection is often cancelled out by thermal advection. Likewise, vorticity advection can negate the anticipated effects of thermal advection. $\vec{Q}$ provides a quantitative way of measuring both elements. They cannot be easily computed by hand, but certain software programs and weather analysis systems can easily calculate the values. In the next chapter, we will apply quasigeostrophic theory to mid-latitude synoptic-scale weather analysis.

**7.4: The Equations of Motion for QG Theory**

The equations of motion suitable for QG theory can be expressed in terms of the vorticity equation and the thermodynamic equation. We will now apply the assumptions of QG theory to these equations. We will develop a pair of prognostic equations (i.e. PDEs that predict the evolution of atmosphere) and a pair of diagnostic equations (i.e. PDEs that diagnose the instantaneous state of the atmosphere).

**7.4.1: The QG Vorticity Equation**

Recall that that the vorticity equation can be written approximately as

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} + v \frac{\partial \zeta}{\partial y} = -f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$

If we assume that the horizontal advection is accomplished by the geostrophic wind and that the relative vorticity can be described by the geostrophic relative vorticity, we have

$$\frac{\partial \zeta_g}{\partial t} + u_g \frac{\partial \zeta_g}{\partial x} + v_g \frac{\partial \zeta_g}{\partial y} = -f \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)$$
Using the continuity equation, we have

\[
\frac{\partial \zeta_g}{\partial t} = -u_g \frac{\partial \zeta_g}{\partial x} - v_g \frac{\partial \zeta_g}{\partial y} + f \frac{\partial \omega}{\partial p}
\]

This is known as the **QG vorticity equation** and it states that the local rate of change of geostrophic vorticity is given by the geostrophic vorticity advection and divergence (or convergence).

**7.4.2: The QG Thermodynamic Equation**

From Chapter 6, we can write the thermodynamic energy equation as

\[
c_p \frac{DT}{Dt} - \alpha \frac{DP}{Dt} = \dot{Q}
\]

Expressing the thermodynamic energy equation in isobaric coordinates gives

\[
\left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) - \sigma_p \omega = \dot{Q} \frac{\dot{Q}}{c_p}
\]

where

\[
\sigma_p = \left( \frac{\alpha}{c_p} - \frac{\partial T}{\partial p} \right)
\]

From the hydrostatic equation in isobaric coordinates, we see that

\[
T = -\frac{p}{R} \frac{\partial \Phi}{\partial p}
\]

The thermodynamic energy equation can be rewritten as

\[
-\frac{p}{R} \left[ \frac{\partial}{\partial t} (\partial \Phi / \partial p) + u \frac{\partial}{\partial x} (\partial \Phi / \partial p) + v \frac{\partial}{\partial y} (\partial \Phi / \partial p) \right] - \sigma_p \omega = \dot{Q} \frac{\dot{Q}}{c_p}
\]

If we approximate that the horizontal wind as geostrophic and assume that the diabatic heating is negligible, then we obtain the **QG thermodynamic equation**.
This expression states that the local rate of temperature (or thickness) is given by the thermal advection and adiabatic warming/cooling by vertical motions

**7.4.3: The QG Omega Equation**

A diagnostic equation for synoptic-scale vertical motions arises from considering the QG vorticity and QG thermodynamic equation. Note that the geostrophic relative vorticity can be written as

\[
\zeta_g = \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} = \frac{1}{f} \left( \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} \right) = \frac{1}{f} \nabla^2 \Phi
\]

Therefore, the QG vorticity equation and the QG thermodynamic equation can be written in terms \( \partial \Phi / \partial t \):

\[
\frac{1}{f} \nabla^2 \left( \frac{\partial \Phi}{\partial t} \right) = -\vec{V}_g \cdot \nabla \zeta_g + f \frac{\partial \omega}{\partial p}
\]

\[-\frac{\partial}{\partial p} \left( \frac{\partial \Phi}{\partial t} \right) = -\vec{V}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) + \sigma_p \omega
\]

Eliminating \( \partial \Phi / \partial t \) between both equations produces a diagnostic equation for \( \omega \)

\[
\sigma_p \left( \nabla^2 + \frac{f^2}{\sigma_p \partial p^2} \right) \omega = f \frac{\partial}{\partial p} \left[ \vec{V}_g \cdot \nabla \zeta_g \right] + \nabla^2 \left[ \vec{V}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right] \equiv F_1 + F_2
\]

The above equation is known as the **QG omega equation**. First, note that only derivatives in space exist in space for the QG omega equation so that it is a *diagnostic* equation for \( \omega \) in terms of the *instantaneous* geopotential height field. The value of such an expression is that we can obtain from it a measure of \( \omega \) that is not dependent on accurate observations of the wind. It’s a rather complicated-looking expression so we will need to consider what physical meaning the mathematics contains.

The term on the left hand side of the QG omega equation, despite its complicated-looking nature, is essentially a 3-D Laplacian term. If we assume that the vertical motion field displays a sinusoidal vertical profile (which turns out to be a very solid assumption), then \( \partial^2 \omega / \partial p^2 \propto -\omega \). Also, since the Laplacian is a second-derivative operator, a local maximum (minimum) in \( \nabla^2 \omega \) implies a local minimum (maximum) in \( \omega \) itself. Thus, whenever the right hand side of the QG
omega equation is found to be positive (negative), then $\nabla^2 \omega$ is positive (negative), implying that $\omega$ is negative (positive), corresponding to upward (downward) vertical motion. Thus, the QG omega equation can be written approximately as

$$-\omega \propto f \frac{\partial}{\partial p} \left[ \vec{V}_g \cdot \nabla \zeta_g \right] + \nabla^2 \left[ \vec{V}_g \cdot \vec{\nabla} \left( -\frac{\partial \Phi}{\partial p} \right) \right]$$

$F_1$ physically represents the vertical derivative ($-\partial/\partial p$) of geostrophic vorticity advection ($-\vec{V}_g \cdot \nabla \zeta_g$). Thus, if an environment is characterized by geostrophic cyclonic vorticity advection increasing (decreasing) with height, then this term is positive (negative) implying that the environment will be characterized by upward (downward) vertical motion. Recall the schematic in Figure 7.6. Since geostrophic vorticity near the surface high and low is concentrated at those locations and the circulations are nearly closed at that level, the geostrophic vorticity advection near the surface is rather small. Geostrophic vorticity advection above the surface low, however, is large and positive so that that column experiences upward-increasing cyclonic vorticity advection and, hence, upward vertical motion. Geostrophic vorticity advection above the surface high is large and negative and that column experiences upward-decreasing cyclonic vorticity advection and, hence, downward vertical motion. It’s important to note that since geostrophic vorticity is proportional to $\Phi$ when the column above the surface low experiences a greater increase in geostrophic vorticity aloft than near the surface, this implies that $\nabla^2 \left[ \partial \Phi'/\partial t \right] > 0 \Rightarrow \partial \Phi'/\partial t < 0$. In order to experience the requisite thickness decrease, the column must cool. The cooling is achieved by adiabatic expansion of the rising air.

$F_2$ describes the Laplacian of horizontal temperature advection. If an environment is characterized by a local maximum in warm (cold) air advection, then this terms is positive (negative), corresponding to upward (downward) vertical motion. It is important to note that, according to QG omega equation, warm (cold) air advection alone is not enough to diagnose the sense of the vertical motion – it is the Laplacian of the temperature advection that matters. This implies that only heterogeneity in the thermal advection field is associated with $\omega$. In other words, if the gradient of horizontal temperature advection is zero (i.e. there is uniform horizontal temperature advection) then the whole term will be zero and no vertical motion will be forced. The vertical motion fields described by the quasi-geostrophic omega equation are precisely those vertical motions that are required to keep the thermal and mass fields in hydrostatic and geostrophic balance. These diagnosed vertical motions also tend to be an accurate description of the large-scale vertical motions observed in the mid-latitude atmosphere.
7.4.4: The QG Height Tendency Equation

Recall that the QG vorticity and thermodynamic equations were given by

\[
\frac{\partial}{\partial t} \left( -\frac{\partial \Phi}{\partial p} \right) = -u_g \frac{\partial}{\partial x} \left( -\frac{\partial \Phi}{\partial p} \right) + v_g \frac{\partial}{\partial y} \left( -\frac{\partial \Phi}{\partial p} \right) + \sigma_p \omega \\
\frac{\partial \zeta_g}{\partial t} = -u_g \frac{\partial \zeta_g}{\partial x} - v_g \frac{\partial \zeta_g}{\partial y} + \frac{f}{\partial p} \frac{\partial \omega}{\partial p}
\]

If we represent the geopotential tendency as \( \chi = \partial \Phi / \partial t \), then the geostrophic vorticity tendency can be expressed as

\[
\frac{\partial \zeta_g}{\partial t} = \frac{1}{f} \nabla^2 \chi
\]

and the above two expressions can be rewritten as

\[
\nabla^2 \chi = -f \vec{V}_g \cdot \nabla \left( \frac{1}{f} \nabla^2 \Phi \right) + f^2 \frac{\partial \omega}{\partial p}
\]

\[
\frac{\partial \chi}{\partial p} = -\vec{V}_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) - \sigma_p \omega
\]

Eliminating \( \omega \) between both equations produces a diagnostic equation for \( \chi \):

\[
\left( \nabla^2 + \frac{f^2 \frac{\partial^2}{\sigma_p \partial p^2}} \right) \chi = -f \vec{V}_g \cdot \nabla \left( \frac{1}{f} \nabla^2 \Phi \right) - \frac{f^2}{\sigma_p \partial p} \left[ \vec{V}_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) \right]
\]

This expression is known as the **QG height tendency equation**. The operator on the left hand side is exactly the same as the operator on the left hand side of the QG omega equation and can be interpreted similarly. Therefore

\[
-\chi \propto -f \vec{V}_g \cdot \nabla \left( \frac{1}{f} \nabla^2 \Phi \right) - \frac{f^2}{\sigma_p \partial p} \left[ \vec{V}_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) \right] \equiv F_3 + F_4
\]

\( F_3 \) describes the effect of geostrophic vorticity advection on height falls. Recall that Figure 7.7 shows a schematic upper-tropospheric trough with a cyclonic vorticity maximum at its base. Immediately to the east (west) of the trough axis, there is positive (negative) geostrophic vorticity advection. In the absence of other processes, PVA is associated with height falls (\( \chi < 0 \)) while NVA is associated with height rises (\( \chi > 0 \)). Interestingly, however, since the
geostrophic vorticity advection is precisely zero at the axis of the trough (since the gradient of
geostrophic vorticity is zero there), there is no height tendency at that point. Since that point
represents the location of lowest geopotential height in the first place, we see that the geostrophic
vorticity term can only propagate an already existing disturbance – it cannot intensify it.

\[ F_4 \] can be rewritten, using the hydrostatic equation, as

\[ -\frac{f^2}{\sigma_p} \frac{\partial}{\partial p} \left[ -\vec{v}_g \cdot \nabla \left( -\frac{\partial \Phi}{\partial p} \right) \right] = \frac{Rf^2}{\sigma_p p} \left[ -\frac{\partial}{\partial p} \left( -\vec{v}_g \cdot \nabla T \right) \right] \]

which is recognized as the vertical derivative of geostrophic temperature advection. We see that
geostrophic temperature advection increasing (decreasing) upward is associated with height falls
(rises).

7.5: Movement of Weather Systems

Here, we will use QG theory to understand the movement of upper-level and surface weather
systems.

7.5.1: Movement of Upper-Level Systems

The apparent movement of pressure and height systems is due to the horizontal gradient
of the isallobaric field. Systems that move in the same direction as the background flow are said
to progress, while systems that move in the opposite direction to the background flow are said to
retrograde. In a wavetrain of short waves there is PVA and geopotential height falls
downstream from troughs and upstream from ridges, and NVA and geopotential height rises
downstream from ridges and upstream from troughs according to QG theory, as shown in Figure
7.11. In a wavetrain of long waves, there is PVA and geopotential height falls downstream from
troughs in the easterlies, upstream from ridges in the easterlies (Figure 7.11), downstream from
ridges in the westerlies, and upstream from troughs in the westerlies (Figure 7.11). There is NVA
and geopotential height rises downstream from ridges in the easterlies, upstream from troughs in
the easterlies (Figure 7.11), downstream from troughs in the westerlies, and upstream from
ridges in the westerlies (Figure 7.11). The effect of vorticity advection on a wavetrain of short
waves is to make a progress along with the background flow (the background flow is thought of
as the carrier of short-wave troughs), while the effect of vorticity advection on a wavetrain of
long waves is to make it retrograde if the background flow is easterly.
When a progressing short-wave trough in the westerlies approaches a stationary long-wave trough or a short-wave trough embedded equatorward in weaker westerlies, the effective horizontal wavelength decreases and the stationary trough may become progressive (Figure 7.12) as geostrophic-vorticity advection becomes significant. It is said that the upstream trough has **kicked out** the stationary trough. The upstream trough is therefore referred to as the **kicker**.

The wind speeds in the vicinity of a trough (of limited latitudinal extent) are often indicators of the motion of the trough:
- When the strongest winds are upstream from the trough, the trough tends to "dig" equatorward.
- When the strongest winds are downstream from the trough, the trough tends to "lift out" poleward.
These observations can be explained in terms of QG theory. Assume that the variations in wind speed in the vicinity of the trough are approximately geostrophic. If the region of maximum wind speed is upstream from a trough in the westerlies, then the geostrophic-vorticity maximum associated with the trough is also upstream from the trough. The relative maximum in geostrophic vorticity is located between the area of greatest cyclonic curvature, which is along the trough axis, and the area of greatest cyclonic shear, which is on the poleward side of the upstream wind maximum (Figure 7.13). The region of maximum cyclonic vorticity advection and height falls is therefore near the “base” of the trough axis, not east of it. The trough therefore has an equatorward component of motion. Similarly, when the region of maximum wind speed is downstream from a trough in the westerlies, the geostrophic-vorticity maximum associated with the trough is also downstream from the trough: The relative maximum in geostrophic vorticity is located between the area of greatest cyclonic curvature along the trough axis, and the area of greatest cyclonic shear, on the poleward side of the downstream wind maximum (Figure 7.13). The region of maximum PVA and height falls is therefore downstream and poleward from the “base” of the trough. The trough therefore has a poleward component of motion.
Having discussed certain aspects of the motion of systems, it's appropriate to consider a phenomenon characterized by the lack of motion. A **blocking pattern** exists when systems do not progress at all within the latitude belt of the baroclinic westerlies. The zonal movement of short waves is effective halted, as the zonal current velocity vanishes, and hence vorticity advection, the major mechanism by which upper level systems move, become negligible. Blocking patterns make life miserable for many people affected by them, since those living near upper-level cyclones tend to experience a persistent combination of precipitation and relatively cool temperatures, while those living near upper-level anticyclones tend to experience drought conditions. Blocking situations are frequently accompanied by extreme meteorological events. The three most common types of blocking patterns are: (1) High-over-low block; (2) Omega block; and (3) Stationary, high-amplitude ridge.
In the Northern Hemisphere the high-over-low block (Figure 7.14) occurs most frequently over the west coasts of Europe and North America. This type of a block is often referred to as split flow, since the basic current “splits” around the block. Owing to the geographical preference for this type of block, horizontal variations in surface heating and/or topography probably play an important role in creating the block. The flow pattern in the omega block has the shape of the capital letter Ω. It is a zonally oriented configuration of a high sandwiched in between two lows. The third type of block, the stationary, high-amplitude ridge (Figure 7.19) is associated mainly with hot, dry, dull weather.
7.5.2: Movement of Surface Pressure Systems

In Chapter 3, we discussed the kinematics of the pressure field: A surface high-pressure area (i.e. an anticyclone) moves from a region of pressure falls toward a region of pressure rises, that is, in the direction of the isallobaric gradient. Previously, we argued that a region of pressure falls at the surface is also a region of height falls near the ground and convergence. (Vorticity advection is neglected because surface systems are usually relatively circular and thus are associated with little vorticity advection.) Similarly, a region of pressure rises at the surface is also a region of height rises near the ground and divergence. On a level surface a region of convergence is accompanied by rising motion aloft owing to continuity, and a region of divergence is accompanied by sinking aloft. Therefore, the following rules govern the motion of surface pressure systems:

- A surface anticyclone moves away from a region of rising motion toward a region of sinking motion.
- A surface cyclone moves away from a region of sinking motion toward a region of rising motion.

According to QG theory, surface anticyclones on level surfaces have the following motion characteristics:

- From regions of geostrophic absolute vorticity advection becoming more cyclonic with height toward regions of geostrophic absolute vorticity advection becoming more anticyclonic with height;
- From regions of geostrophic warm advection toward regions of geostrophic cold advection;
- From regions of diabatic heating toward regions of diabatic cooling.

By similar reasoning, cyclones on level surfaces have the following motion characteristics:

- From regions of geostrophic absolute vorticity advection becoming more anticyclonic with height toward regions of geostrophic absolute vorticity advection becoming more cyclonic with height;
- From regions of geostrophic cold advection toward regions of geostrophic warm advection;
- From regions of diabatic cooling toward regions of diabatic heating.

The actual movement of cyclones and anticyclones over a level surface is determined by the sum of these effects, which in turn is controlled by the geometry of the height field. There is usually cold advection equatorward and eastward of, and warm advection poleward and westward, of surface anticyclones. Thus the effect of temperature advection is usually to “move”
surface anticyclones toward the southeast (in the Northern Hemisphere). Since surface anticyclones often form upstream from a trough and downstream from a ridge in the westerlies, the flow aloft is also toward the northeast. There is usually warm (cold) advection equatorward and eastward (poleward and westward) of surface anticyclones. Thus the effect of temperature advection is usually to “move” surface cyclones toward the northeast (in the Northern Hemisphere). Since surface cyclones often form downstream from a trough and upstream from a ridge in the westerlies, the flow aloft is also toward the northeast.

The effects of differential vorticity advection cannot be as easily generalized. If the surface low is co-located with the region of maximum vorticity advection becoming more cyclonic with height, then the cyclone will deepen, but will not move as a result of differential vorticity advection alone. The effect of differential vorticity advection is to slow down the eastward component of motion if the region of maximum vorticity advection becomes more cyclonic with height is located upstream (downstream) aloft from the surface low. It's apparent from the geometry of the 3D height field associated with a typical cyclone or anticyclone that the apparent motions of surface cyclones and anticyclones to a large extent follow the flow aloft. The tendency of surface pressure systems to follow the flow aloft is called steering. However, surface cyclones and anticyclones are not solid objects being carried along by the flow aloft, and their movement is really by propagation of the low- and high-pressure centers.
Chapter 8: Jets and Jet Streaks

Just as it is curious that the temperature field has regions of concentrated gradients (i.e. frontal zones), it is also fascinating that the wind field has regions of concentrated wind. The discovery of the jet stream during the 1940s by a combination of researchers and forecasters was perhaps as important as the discovery of the surface front over 25 years earlier. The advent of the rawinsonde network, the increased use of high-altitude aircraft, and an effort to investigate the synoptic conditions accompanying severe-storm outbreaks all provided the impetus for the study of jets.

Figure 8.1 300 mb wind speed and geopotential height analysis for 9 April 2007 at 00Z.

A jet (or jet stream) is an intense, narrow, quasi-horizontal current of wind that is associated with strong vertical wind shear. The qualifiers “intense” and “narrow” are somewhat subjective, however. Typically, jet streams range from 500 – 6000 km in length and 100 – 400 km in width. A jet streak is a localized maximum of wind speed embedded within a jet stream. A jet stream can refer to one of two features:
• A climatological maximum in the zonal wind. The temporal duration of the climatology can vary from on the order of one week to on the order of one year.
• A localized corridor of high wind speed at one given analysis time. This manifestation of a jet stream is that which is typically analyzed on synoptic-scale weather maps and what which we will consider within this class.

In the context of this course, we will primarily focus upon upper tropospheric manifestations of jet streams since they primarily influence the motion and evolution of synoptic-scale systems and contribute to the initiation and evolution of mesoscale systems and deep convection. We will only cursorily discuss the low-level jet, a lower tropospheric manifestation of a jet stream. Likewise, we will focus primarily upon jet streams located within the middle and higher latitudes.

8.1: The Polar Jet

The polar jet (sometimes referred to as the polar-front jet) is located along the tropopause. On isobaric maps, this typically can be seen on between 200 or 300 mb charts. The winds in this jet are westerly (i.e. blow from west to east) and often exceed 85 m/s. It is present year round and migrates north (∼50°N) during the summer months and migrates south (∼35°N) during the winter months. It is associated with strong quasi-horizontal temperature gradients in the lower troposphere and strong vertical wind shear. These features are often associated with the polar front, the name given to cold fronts that trail cyclones in polar front theory.

The link between the polar jet and the aforementioned quasi-horizontal temperature gradients is manifest through the thermal wind relationship. Winds along the polar jet typically have a westerly component to them, as shown in Figure 8.2; concordantly, given westerly vertical wind shear and thus a westerly thermal wind, relatively cold lower tropospheric air is found poleward of the polar jet and relatively warm lower tropospheric air is found equatorward of the polar jet, as shown in Figure 8.3. Meridional undulations of the jet stream are associated with poleward and equatorward displacements of warm and cold air, respectively.

Figure 8.2 Schematic of westerly jet caused by the Coriolis force
Presuming that the polar jet is a westerly jet, the height of the tropopause is lower poleward and higher equatorward of the polar jet. This can be viewed in the context of thickness arguments, with greater (lesser) tropospheric depth where it is relatively warm (cold). Climatologically the height of the tropopause gently slopes upward from the pole to the equator through the polar jet.

![Figure 8.3 Schematic of jet stream in thermal wind balance](image)

A vertical cross-section through the polar jet and associated polar front is depicted in Figure 8.4. The polar front slopes northward with height, from south of Norman, OK (OUN) toward North Platte, NE (LBF), as indicated by the sloping corridor of tightly-packed isentropes. Winds are out of the north beneath the polar front and primarily out of the west-southwest ahead of and atop the polar front. The polar jet is centered at around 300 hPa at Dodge City, KS (DDC) and OUN and has maximum wind speeds of 140-150 kt at the height of the tropopause. The tropopause itself is higher to the south, in the warm air, and lower to the north, in the cold air. The tropopause is locally depressed downward, or folded, at its intersection with the polar front. Note the relationship between the polar jet and the lower tropospheric polar cold frontal zone:
the polar jet is found atop the cold frontal zone, as we would expect from thermal wind balance. This emphasizes the point that the basic forcing mechanism of all jet streams (including the polar jet stream) would be a strong low-level temperature gradient, as illustrated in Figure 8.5. In Figure 8.5, notice how all of strong upper-level jets (at 300 mb) are located directly above a strong low-level temperature gradient (at 850 mb).

Figure 8.4 Vertical cross-section of potential temperature (contoured every 5 K) and wind (half-barb: 5 kt, full barb: 10 kt, pennant: 50 kt) through a strong polar front (blue annotation) and polar jet (red annotation) system. The thicker black line denotes the approximate location of the tropopause. The northern extent of the cross-section, at left, is at Glasgow, MT. The southern extent of the cross-section, at right, is at Del Rio, TX. Reproduced from *Synoptic-Dynamic Meteorology in Midlatitudes (Vol.II)* by H. Bluestein, their Figure 2.86.
Figure 8.5 (Left) 300 mb wind speed and geopotential height analysis for 9 April 2007 for 00Z. (Right) 850 mb temperature and geopotential height analysis for 9 April 2007 for 00Z.

8.2: The Subtropical Jet

Figure 8.6 Schematic of the relative locations of the polar jet and the subtropical jet

Like the polar jet, the subtropical jet is located along the tropopause. On isobaric map, this can be typically observed on 200 mb charts. The subtropical jet is primarily a wintertime
phenomenon found between 20 – 35°N/S latitude, as shown in Figure 8.6. Mean wind speeds in excess of 70 m/s are found southeast of Japan. High wind speeds are found along the east coast of North America slightly north of 30°N, and in a belt extending from North Africa through Northern India. Weak cyclonic curvature in the mean is found in subtropics off the west coast of the United States and off the west coast of Africa. In the time-mean view of the subtropical jet, it can have the appearance of being a continuous jet around the globe within the subtropics. Consequently, the subtropical jet can be viewed as a quasi-steady or quasi-persistent feature of the cold season climatology. Owing to steadiness and large-scale nature of the subtropical jet, it appears as if planetary-scale processes must play an important role in its maintenance. On a day-to-day basis, however, the subtropical jet may merge with or become indistinct from the polar jet, particularly when the latter protrudes equatorward.

A vertical cross-section through the subtropical jet is depicted in Figure 8.7. Unlike the polar jet, there is no well-defined frontal boundary that slopes toward the cold air with height in association with the subtropical jet. The meridional temperature gradient associated with the vertical wind shear underneath the subtropical jet (again through the thermal wind relationship) is largely confined to a shallow vertical layer within the middle to upper troposphere. The

Figure 8.7 As in Figure 8.4, except for a south-to-north (left-to-right) vertical cross-section through the subtropical jet. Here, the blue annotation reflects the region of largest horizontal temperature gradients rather than a true frontal zone. Reproduced from Synoptic-Dynamic Meteorology in Midlatitudes (Vol. II) by H. Bluesetein, their Figure 2.92.
The subtropical jet itself is centered at around 200 hPa at Waycross, GA (AYS) and has maximum wind speeds of 120-125 kt at the height of the tropopause. The tropopause itself, though not depicted on this figure, is higher to the south and lower to the north.

A band of cirrus clouds is often seen on the anticyclonic-shear side of the subtropical jet. Smaller transverse bands are sometimes seen within this main band. The cirrus may be generated as air rises downstream from a trough in the tropics, and is subsequently transported downstream by the subtropical jet. The sharp poleward edge of the cirrus owes its existence to deformation and a horizontal moisture gradient. There is some observational evidence that, although severe convective storms sometimes develop equatorward of the subtropical jet and its cirrus band, most develop poleward of the subtropical jet. The reason for this is not known; however, it is possible that radiative effects from the cirrus may play an important role in suppressing severe convection under the cirrus.

Like the Polar-Front jet, the subtropical jet is associated with a change in height of the tropopause. The meridional temperature gradient associated with the vertical shear under the subtropical jet is concentrated in a shallow layer, not in a relatively deep layer as the Polar-Front jet. Sometimes the subtropical jet is associated with a portion of the Polar-Front jet that has migrated equatorward into the subtropics and has lost its low-level baroclinicity.

The presence of the subtropical jet can be viewed from two somewhat complementary perspectives. The first relates to the Hadley and Ferrel general circulation cells, as shown in Figure 8.8. The Hadley cell, the meridional vertical circulation that characterizes the tropical latitudes, is associated with poleward flow in the upper troposphere and equatorward flow in the lower troposphere. The Ferrell cell, the meridional vertical circulation that characterizes the mid-latitudes, is associated with equatorward flow in the upper troposphere and poleward flow in the lower troposphere. This promotes convergence aloft and divergence at the surface, acting to

![Figure 8.8 Schematic of general circulation cells associated with subtropical jet.](image-url)
intensify the meridional temperature gradient aloft and weaken it at the surface. The latter explains why there is often little or no frontal structure at the surface beneath the subtropical jet. From thermal wind balance, the stronger meridional temperature gradient aloft, with warmer air toward the equator and colder air toward the poles, gives rise to the westerly subtropical jet.

Figure 8.9 Schematic of angular momentum conservation associated with subtropical jet.

The second relates to the conservation of absolute angular momentum in the context of the Hadley cell, as shown in Figure 8.9. Absolute angular momentum is a function of both the rotation of the Earth and the velocity of the air parcel; following the motion, in the absence of friction, an air parcel generally strives to conserve absolute angular momentum. Consider an air parcel that is initially at rest at the Equator within the upper troposphere. If this parcel is displaced poleward from the Equator, such as within the poleward flow aloft found with the Hadley cell, it will accelerate (to ~100-200 kt) in order to conserve absolute angular momentum. It will also deflect toward a westerly flow due to the effects of the Coriolis force, with rightward deflection in the northern hemisphere and leftward deflection in the southern hemisphere, thus giving rise to the subtropical jet.
8.3: The Low-Level Jet

Several types of low-level jets have been identified in the United States. There are two types of low-level jets (or LLJs) that are of particular interest:

- Nocturnally-driven LLJs
- LLJs induced by upper tropospheric and/or synoptic-scale forcing.

Nocturnally-driven LLJs are at their maximum intensity at night; there may be little to no evidence of a nocturnal LLJ during the daytime hours. The jet is often located just east of a lee trough and may be located on a surface map where sustained winds are strongest (and the gusts are strongest). They owe their existence to sloping topography, such as is found across the Great Plains of the United States (with higher terrain to the west gradually sloping downward toward the east), and to inertial oscillations. Nocturnal LLJs are typically found at or just above the surface. In the Great Plains, they are typically associated with southerly winds greater than 15 m/s and may influence thunderstorm development, maintenance, and upscale growth, as shown in Figure 8.10

The Southern Plains topographic low-level jet is located at an average height of 800 m. It is often responsible for the rapid advection northward of moisture from the Gulf of Mexico. Undergoing a marked diurnal variation in strength, this low-level jet is strongest at night and weakest during the day. The observed nocturnal maximum in thunderstorm activity over the Plains might be a result of the increase in moisture advection into thunderstorms at cloud-base level at night owing to the low-level jet. The jet is often located just east of a lee trough and may be located on a surface map where sustained winds are strongest (and the gusts are strongest).

LLJs induced by upper-tropospheric and/or synoptic forcing are often found in association with upper tropospheric jets and synoptic-scale cyclones. The conveyor belts of a mature midlatitude synoptic-scale cyclone that we will examine in a future chapter can be viewed, to first approximately, as manifestations of LLJs, as shown in Figure 8.11. The presence of such LLJs is not dependent upon the time of day; rather they can appear at all times of the day. Likewise, these LLJs can be every bit as strong as their nocturnal counterparts. As compared to nocturnal LLJs, however, LLJs induced by other forcing are typically found at slightly higher altitudes (~850 mb).

It should be noted that the two types of LLJs are not mutually exclusive of one another; in the presence of synoptic-scale forcing, a given southern LLJ over the Great Plains may have structural characteristics of both types of LLJs.
Figure 8.10 Schematic of nocturnal LLJ. Intensity fluctuations of LLJs are linked to diurnal changes in the low-level temperature gradients along the gradually sloping east-west topography.

Figure 8.11 Schematic of pre-frontal LLJ. These are located just ahead (east) of strong cold fronts and are responsible for the rapid advection of warm moist air that can help initiate deep convection along the front.

8.4: Jet Streaks

Jet streaks are local maxima of wind speed in the jet stream. They are typically 250 – 1000 km in length and 50 – 200 km in width. Jet streaks migrate and evolve over time scales from a few hours to a few days and their motion are often much slower than the speed of the wind within the jet stream. For the sake of our course, the jet streaks contribute to the evolution of synoptic-scale systems since most contain strong positive vorticity advection (PVA).
8.4.1: The Straight Jet Model

We first consider an idealized view of an upper-tropospheric jet streak, one that is associated with primarily “straight” (or non-curved flow) through the jet itself, as depicted in Figure 8.12. Before we proceed, let’s recall some minor terminology notes. The jet entrance region is the region where the wind accelerates. Conversely, the jet exit region is the region where the wind decelerates. Likewise the “right” and “left” sides of the jet are defined as to the right and left of the jet with your back to the wind. Thus, for a westerly jet streak, the right side of the jet streaks is to the south and the left side of the jet streak is to the north.

![The Four Quadrant Straight Jet Model](image)

Figure 8.12 Schematic of straight jet streak model. Isotachs are shaded in blue for a westerly jet streak (single large arrow). Thick red lines denote geopotential height contours. Thick black vectors represent cross-stream (transverse) ageostrophic winds with magnitudes given by arrow length. Vertical cross sections transverse to the flow in the entrance and exit regions of the jet (J) are shown in the bottom panels along A-A' and B-B', respectively. Convergence and divergence at the jet level are denoted by "CON" and "DIV". "COLD" and "WARM" refer to the air masses defined by the green isentropes.

In Chapter 6, we stated that following properties of the ageostrophic wind:
- The ageostrophic wind is a measure of the horizontal acceleration in the atmosphere
The ageostrophic wind is perpendicular to the horizontal acceleration vector.

The ageostrophic wind is directed to the left of the acceleration vector in the Northern Hemisphere.

As a parcel enters into the jet, it moves from west to east at a progressively faster rate of speed meaning that it accelerates. Thus, following the flow, $v_{ag}$ (the meridional component of the ageostrophic wind) must be positive, indicating a south-to-north flow across the jet within its entrance region. From mass continuity, this implies upper-tropospheric divergence and middle tropospheric ascent in the right entrance region and upper tropospheric convergence and middle tropospheric descent in the left entrance region.

Conversely, as a parcel exits the jet, it moves west to east at a progressively slower rate of speed, meaning that it decelerates. Thus, following the flow, $v_{ag}$ must be negative, indicating a north-to-south flow across the jet within its exit region. From continuity, this implies upper-tropospheric convergence and middle tropospheric descent in the right exit region and upper tropospheric divergence and middle tropospheric ascent in the left exit region.

In the lower troposphere, the patterns of ascent and descent within the jet entrance and exit regions promote the opposite sense of ageostrophic flow – negative (north-to-south) within the jet entrance region and positive (south-to-north) within the jet exit region. Thus, vertical accelerations exist within the jet entrance and exit regions. As rising motion is found equatorward of the jet streak, where it is relatively warm, in its entrance region, the jet entrance region circulation is known as a thermally-direct circulation. Conversely, as rising motion is found poleward of the jet streak, where it is relatively cold, in its exit region, the jet exit region circulation is known as a thermally-indirect circulation. These systems are referred to as the transverse geostrophic equation.

We can also arrive at the same answer for the vertical circulation pattern from quasi-geostrophic theory. For the straight jet streak situation, a single vorticity maximum occurs on the left side of the jet axis, and a vorticity minimum occurs on the right-hand side, as shown in Figure 8.13. This result follows from the relationship between vorticity and divergence. Given the westerly geostrophic flow present for the straight jet, it immediately follows that positive vorticity advection (PVA) will be present downstream of the vorticity maximum in the left exit region of the jet, as well as the upstream of the vorticity minimum in the right entrance region. Conversely, we find negative vorticity advection (NVA) patterns in the left entrance and right exit regions. Since both the vorticity and the zonal geostrophic wind component increase with height between the ground and the level of the jet, then PVA increases with height in the left exit and right entrance regions; similarly, NVA increases with height in the opposite quadrants. Assuming no thermal advection (i.e. the isotherms are parallel to the winds), then quasi-
geostrophic theory states that rising motion will occur in the two quadrants in which there is a vertical increase of PVA and subsidence will be present in the opposite two quadrants.

**THE FOUR QUADRANT STRAIGHT JET MODEL**

Figure 8.13 Isotachs are shaded in blue for a westerly jet streak (single large arrow). Thick red lines denote vorticity isopleths, with the vorticity maximum (VORT MAX) and minimum (VORT MIN) shown on the left and right sides of the jet core, respectively. Resultant patterns of positive vorticity advection (PVA) and negative vorticity advection (NVA) are also depicted.

**8.4.2: The Inclusion of Jet/Flow Curvature**

To this point, we have considered the distribution of ageostrophic flow for a “straight” jet streak. However, jet streaks and the synoptic-scale flow accompanying them often exhibit at least weak curvature. An example of a weakly curved jet streak is depicted within Figure 8.14. In a qualitative sense, the interpretation is identical to that of the straight jet streak example described in Figure 8.12. In a quantitative sense, however, the interpretation changes slightly. In Figure 8.14 posed below, there is strong synoptic-scale diffluence in the left exit region and strong synoptic-scale confluence in the left entrance region of the jet streak. In the right entrance and exit regions, there is comparatively little synoptic-scale confluence or diffluence.

Figure 8.14 Schematic of a slightly curved upper tropospheric jet streak
Thus, in Figure 8.14, the superposition of the synoptic-scale forcing (from continuity) and that associated with the across-jet vertical circulations enhances descent and ascent in the left entrance and exit regions of the jet streak, respectively. There is minimal impact upon ascent and descent in the right entrance and exit regions of the jet streak, respectively.

8.4.3: Coupling of Jet Streaks and Surface Fronts

Figure 8.15 Schematic of jet streak-front coupling leading to enhanced convection. The left exit or right entrance region is above the front, which helps destabilize the potentially unstable low-level air. This increases the likelihood of deep convection.

Uccellini and Johnson in 1979 first suggested that the transverse vertical circulation associated with an upper-level jet streak could become coupled to the wind field at low levels if the circulation extends down to low levels. Underneath the rising branch of the circulation, at the surface, there is convergence; underneath the sinking branch of the circulation, there is divergence. According to the vorticity equation, vorticity will become more cyclonic and surface pressure will fall under the rising branch, while vorticity will become more anticyclonic and surface pressure will rise under the subsiding branch. This pressure-fall/pressure-rise couplet at the surface in the plane of the circulation is associated with an isallobarically induced ageostrophic wind, which is necessarily perpendicular to the flow in the jet streak if the jet streak is straight. The ageostrophic wind exists because the jet streak “feels” the effect of the surface boundary.
In the exit region of a jet streak in westerly flow, the surface-induced ageostrophic flow is toward the pole, while the ageostrophic flow aloft is toward the equator. With cold air on the poleward side, and warm air on the equatorward side, the vertical circulation destabilizes the atmosphere, owing to differential ageostrophic temperature advection; temperature advection becomes more negative with height (i.e. warm advection below, cold advection aloft). This can lead to convection if there is a supply of low-level moisture on the equatorward side, as shown in Figure 8.15.

![Figure 8.16 Schematic of jet streak-front coupling leading to suppressed convection. The left entrance or right exit region is above the front, which prevents destabilization of the potentially unstable air. This decreases the likelihood of deep convection.](image)

Others have suggested that when the vertical circulation associated with the exit region of an upper-level jet streak is situated right over the vertical circulation associated with a surface front, there is sinking motion over the warm, moist air mass east of the front, and convection is suppressed since the air aloft is being stabilized as shown in Figure 8.16. However, when the circulation of the exit region of the jet streak moves farther east, it becomes coupled to the frontal circulation below, and the air mass is destabilized. The orientation of a surface front, an upper-level jet streak, and a low-level jet streak can further enhance deep convection along the front, as shown in Figure 8.17. A favorable combination of ageostrophic circulations from each jet streak and the surface front can create strong lift along the warm (unstable) side of the front, which often is the location of the most severe deep convection.

![Figure 8.17 Schematic of upper-level jet streak coupling with a surface front and a low-level jet streak.](image)
**8.5: The Dynamics of Jets and Jet Streaks**

Suppose that a jet or jet streak is largely geostrophic (and hydrostatic). Since the formation of a front is accompanied by an increase in thermal-wind shear, the geostrophic wind aloft may increase and form a jet or jet streak. However, it is also instructive to consider the formation of jets and jet streaks in general, which are not necessarily geostrophic, owing to their small cross-stream scale and associated high wind speeds. Furthermore, the low-level jet is usually in the boundary layer, where turbulent mixing is important.

The intensity of a jet flowing in the $x$ direction may be measured by $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. The intensity of a jet streak, within this jet, may be measured by the three dimensional Laplacian. In the center of the core of a jet

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} < 0, \quad \frac{\partial u}{\partial y} = 0, \quad \frac{\partial u}{\partial z} = 0.$$

We can regard the rate of change of $\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ following air parcel motion as a measure of the increase or decrease in the intensity of the jet. We can similarly regard the rate of change of $\nabla^2 u$ following air parcel motion as a measure of the increase or decrease in the intensity of the jet streak. Let us call the functions

$$J = \frac{D}{Dt} (-\nabla^2_x u), \quad J_s = \frac{D}{Dt} (-\nabla^2 u)$$

the jetogenetical functions for jets and jet streaks, respectively, where $\nabla^2_x = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$. They are analogous to the frontogenetical function and are useful for diagnosing frontal processes when only wind data are available. It follows that

$$J = -\nabla^2_x \left( \frac{Du}{Dt} \right) + \nabla^2_x (\vec{V} \cdot \nabla u) - \vec{V} \cdot \nabla (\nabla^2_x u)$$

$$J_s = -\nabla^2 \left( \frac{Du}{Dt} \right) + \nabla^2 (\vec{V} \cdot \nabla u) - \vec{V} \cdot \nabla (\nabla^2 u)$$

However, if $\nabla u$ is uniform in the neighborhood of the parcel, then the advection terms can be neglected, yielding
\[ J = -\nabla_x^2 \left( \frac{Du}{Dt} \right) \]
\[ J_s = -\nabla^2 \left( \frac{Du}{Dt} \right) \]

The equation of motion along the jet or jet streak is
\[ \frac{Du}{Dt} = f v - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u = f v_{ag} + \nu \nabla^2 u \]

where \( v_{ag} \) is the ageostrophic component of the wind. Substituting the above equation into the jetogenetical functions, we see that on the jet axis or at the jet streak (where \( y = 0 \)),
\[ J = -f \nabla_x^2 v_{ag} - \nabla_x^2 (\nu \nabla^2 u) \]
\[ J_s = -f \nabla^2 v_{ag} - \nabla^2 (\nu \nabla^2 u) \]

The jetogenetical function is positive whenever there is a local maximum in the cross-jet component of the ageostrophic wind. Since the ageostrophic wind is perpendicular and to the left of the parcel acceleration vector (in the Northern Hemisphere), “local” parcel accelerations lead to “local” wind maxima. If \(-f \nabla^2 v_{ag}\) is positive somewhere, then by continuity there must be at least one branch of \( v_{ag} \) in the opposite direction above or below. Thus, vertical circulations are associated with the formation of jets and jet streaks. These vertical circulations may be driven by frontogenetically or frontolytically induced geostrophic forcing. Thus, jets may indeed be byproducts of frontal processes. The second term, which involves the variation of the Coriolis parameter with latitude, is usually relatively small. The third term is important for jets in the boundary layer such as the low-level jet. However, a dynamical discussion of this term is beyond the scope of these notes.
Chapter 9: Midlatitude Cyclones

The single most common weather element in the middle-latitudes is the frontal cyclone. As a consequence of this fact, the mid-latitude cyclone has been the subject of scientific scrutiny for well over 200 years. In this chapter we will employ the diagnostic tools and dynamical insights thus far developed and apply them to gain an understanding of the structure, evolution, and underlying dynamics of the midlatitude cyclone life cycle.

![Satellite observation of midlatitude cyclone](image)

Figure 9.1 Satellite observation of midlatitude cyclone

This life cycle consists of various stages. We will pursue our investigation of several of these stages by adopting the perspective that the cyclone is the product of development initiated by finite, identifiable disturbances in the flow, not a manifestation of unstable growth of an infinitesimal perturbation. Consistent with this choice of perspective, we will make use of the quasi-geostrophic diagnostics developed in the previous chapter to consider the dynamics of the cyclogenesis, post-mature, and decay stages of the cyclone life cycle. Examination of the post-mature stage will involve consideration of the structural and dynamical nature of the occlusion. Though research regarding the mid-latitude cyclone stretches back into the eighteenth century, we will begin our investigation by considering the broad structural characteristics of these storms starting with the synthesis of prior observations made manifest in the so-called polar front theory of cyclones.
9.1: General Characteristics of Cyclones

Figure 9.2 Effect of introducing a wave in the momentum field into a zonally oriented bundle of column-averaged isotherms. Light gray lines are undisturbed thickness isopleths of the mean state. Dashed lines are the disturbed thickness isopleths after the meridional motions of the wave (arrows) have distorted them. The thick black line shows a schematic geopotential height line. Note that the resulting thickness wave is a quarter wavelength out of phase with the wave in the geopotential height.

The uneven heating of the spherical Earth results in a pole-to-equator temperature gradient on the planet. As a consequence of the dominance of the thermal wind balance outside of the tropics, such a temperature gradient is manifest as a baroclinic westerly vertical shear at middle latitudes. If we consider the rather hypothetical situation in which the mid-latitude flow is purely zonal and in thermal wind balance, then at some middle or upper tropospheric level the geopotential height lines and isotherms would be everywhere parallel. Imagine that a wave-like perturbation was introduced into this flow and that the speed of the wave exactly equaled the speed of the background zonal flow. In such a case, only the meridional motions associated with the perturbation would be discernible. Those meridional motions would promote warm air advection downstream of the trough axis and cold air advection upstream of the trough axis as shown in Figure 9.2, eventually producing a wave in the thermal field that would lag the wave in the momentum field by one-quarter wavelength. In order for this wave-like perturbation to grow, two conditions must be met: (1) the positive and negative zonal temperature anomalies must become larger, and (2) the kinetic energy associated with the wave motions must increase.
Figure 9.3 Fluids of different densities separated horizontally in a container by a dividing wall (thick black line) at $t = 0$. The white dot represents the height of the center of gravity of the two-fluid system. At $t = t_1$, after the divider has been removed, the height of the center of gravity of the fluid system has been lowered by an amount $\delta z$.

The pole-to-equator temperature gradient represents a horizontal density contrast conceptually analogous to that shown in the left panel of Figure 9.3. If, by some mechanism, the dense fluid ends up beneath the less dense fluid (as shown in the right panel of Figure 9.3) then the center of mass of the fluid system has been reduced and there has been a conversion of some of the initial potential energy into the kinetic energy of the fluid motions involved in the rearrangement. That fraction of the total potential energy that can be converted into kinetic energy is known as the available potential energy (APE). Were our hypothetical wave-like disturbance able to convert the APE of the background zonal baroclinic shear into the kinetic energy of its own motions then the wave-like perturbation would grow at the expense of the basic flow. In such a case, we would designate the background flow as unstable to the introduction of such a disturbance.
Mid-latitude cyclones and anticyclones are wave phenomena. As a result, any regional sea-level pressure analysis, such as the example shown in Figure 9.4, will display an alternating sequence of surface high- and low-pressure disturbances. In order that a surface low- (high-) pressure system remain a region of relative low (high) pressure, air must be extracted from (stuffed into) the atmospheric column above the surface. Thus, an alternating sequence of highs and lows, each associated with sinking or rising air in their respective columns, characterizes a mid-latitude wave train as shown in Figure 9.5(a). Recall that based on simple curvature arguments alone, we know that upward (downward) vertical motions occur downstream of trough (ridge) axes at upper tropospheric levels. Consequently, regions of low (high) geopotential height must be located to the west of the rising (sinking) air columns as shown in Figure 9.5(b). Thus, we know that for developing mid-latitude disturbances, the geopotential height axes tilt westward, into the vertical shear, with increasing height.
Recall that at the mature stage of the mid-latitude cyclone, the low-pressure center is located at the peak of the warm sector. The surface anticyclone lies to the west of the surface cyclone with its center close to the center of minimum temperature at sea level. Now, since the hypsometric equation relates thickness to column-averaged temperature, upper tropospheric geopotential minima (maxima) must lie atop relatively cold (warm) columns. Thus, as shown in Figure 9.5(c), the thermal axes of developing mid-latitude waves tilt eastward with increasing height. Finally, note that since the air is rising through the warm column and sinking through the cold column, developing mid-latitude disturbances are characterized by thermally direct vertical circulations which convert the APE of the background baroclinicity, which is itself manifest in the westerly vertical shear of the large-scale flow, into the kinetic energy of the disturbances.
The fact that the structure of the mid-latitude cyclone results in spontaneous conversion of APE to kinetic energy implies that the background zonal baroclinic shear is, indeed, unstable to certain wave-like perturbations and that mid-latitude cyclones are a primary manifestation of this instability. A more fully developed version of this baroclinic instability theory suggests that disturbances of the scale of mid-latitude short waves (3000 to 4500 km in wavelength), in environments characterized by observed values of vertical shear, are those that exhibit the most efficient growth by this mechanism.

Though elements of the foregoing characteristic vertical structure of cyclones were known in the late nineteenth century, almost no mention was made of the vertical wave structure of cyclones in the NCM. The goal in the subsequent sections will not be to provide a comprehensive review of the theory and supporting observations regarding the various stages of the mid-latitude cyclone life cycle, but instead to demonstrate that the diagnostic tools we have developed thus far can be gainfully employed in developing an understanding of the basic elements of that life-cycle evolution.

### 9.2: The Norwegian Cyclone Model

Much of the understanding of mid-latitude cyclones that existed before the turn of the twentieth century was fragmentary and lacked an organizing conceptual framework. Just after the end of World War I, meteorologists at the University of Bergen in Norway, under the leadership of Vilhelm Bjerknes, developed the polar front theory of the structure and life cycle of mid-latitude cyclones, now known colloquially as the **Norwegian Cyclone Model (NCM)**. The essential genius of this conceptual model, which represented a grand synthesis of prior insights concerning the cyclone, was that it described the instantaneous structure of the cyclone while placing that structure into an identifiable life cycle. At the conceptual heart of the NCM was the existence of a globe-girdling, tropospheric deep, knife-like boundary known as the **polar front** which separated cold polar air from warm tropical air (Figure 9.6a). For reasons that were not discussed in the seminal paper by Bjerknes and Solberg (1922) that introduced the NCM, perturbation vortices occasionally developed along this polar front (Figure 9.6b). The existence of such vortices would then serve to deform the polar front, locally ushering tropical air poleward and polar air equatorward (Figure 9.6b). The precise mechanism by which the perturbation vortex would grow in intensity is not well explained in the NCM, but the continued growth of the perturbation was thought to lead to further deformation of the polar front (Figure 9.6c) and a lower sea-level pressure at the center of the perturbation.
Figure 9.6 Evolution of a midlatitude cyclone according to the Norwegian cyclone model. (a) The polar front as a background state. (b) The initial cyclonic perturbation. (c) The mature stage. (d) The occluded stage. The thin solid lines are isobars of sea-level pressure and the arrows are surface wind vectors.

By this so-called mature stage of the life cycle, the deformation of the polar front had become so extreme as to lend the cyclone its now familiar characteristic frontal structure: a cold front extending equatorward and a warm front extending eastward from the sea-level pressure minimum. The region of homogeneous temperature between the two fronts was deemed the warm sector. Continued intensification of the cyclone compelled the cold front to encroach upon, and subsequently overtake, the warm front. Two important results of this process were that (1) the sea-level pressure minimum was removed from the peak of the warm sector and (2) an occluded front developed to connect the cyclone center to the peak of the warm sector (Figure 9.6d).
It was thought that this process could result in the development of two varieties of occluded fronts in cyclones. One of these was the so-called warm occlusion in which the cold front would ascend the warm front upon overtaking it, leading to a vertical structure similar to that portrayed in Figure 9.7(a). Conversely, a so-called cold occlusion would result if the encroaching cold front was able to undercut the warm front and a vertical structure similar to that portrayed in 9.7(b) would result. The warm (cold) occlusion was thought to occur when the air poleward of the warm front was more (less) dense than the air west of the cold front. Note that in either case, the development of the occluded front was associated with the denser air lifting the less dense air aloft. In so doing, the horizontal density contrast originally characterizing the cyclone (manifest in the horizontal temperature gradient associated with the polar front) was reduced and a stable vertical stratification near the cyclone center was gradually put in place. As illustrated in Figure 9.3, transformation of an originally horizontal density contrast into a purely vertical one reduces the center of gravity of a fluid system gradually driving the system to its lowest potential energy state. Based upon this type of energetics argument, the NCM proposed that the development of the occluded front heralded the post-mature phase for a mid-latitude cyclone, a cessation of intensification, and the commencement of cyclone decay. The nature of the cyclone decay was not described in the NCM beyond mention of the fact that the post-mature
phase cyclone would eventually succumb to frictional dissipation associated with the surface of the Earth.

The NCM accounted for the typical cloud and precipitation distribution associated with a mid-latitude cyclone with reference to the vertical structure of the fronts themselves. The cold front was described as a steeply sloped boundary between polar and tropical air masses that steadily advanced into the tropical air. The advance produced upgliding motions along the boundary itself and, as a consequence of its steep slope, the updrafts were vigorous and horizontally restricted leading to a narrow, sometimes squally precipitation distribution (Figure 9.8a). The warm front, on the other hand, was a less steeply sloped boundary between advancing tropical air and gradually retreating polar air (Figure 9.8b). The upgliding motions along the warm frontal surface were considered to be less intense as a consequence of the shallower slope. As a result, the cloudiness associated with the warm front was more horizontally widespread and the precipitation more benign.

Despite its great insights, the NCM, like all great conceptual leaps, certainly has its limitations. For instance, the nature of, and relationship between, the perturbations which grow into cyclones and the large-scale environment that promotes such growth are not addressed in the NCM and yet are clearly at the heart of understanding the mid-latitude cyclone life cycle. In addition, much more dynamically compelling arguments exist for explaining the production of vertical circulations at fronts. The fronts themselves, in fact, are not knife-like discontinuities as suggested by the NCM but zones of contrast across which temperature, density, and pressure are continuous. Neither are the frontal zones continuous through the depth of the troposphere. Note also that the NCM does not describe the physical mechanisms by which the cyclone intensifies from its incipient stage (Figure 9.6b) through its post-mature stage (Figure 9.6d). Also the
development of the surface occluded front, though partly described through reference to the
vertical frontal structure, is more correctly viewed as part of a process of occlusion that needs to
be more comprehensively considered. Additionally, the decay of the cyclone is barely discussed
in the NCM and yet clearly represents a major component of the cyclone life cycle. Finally,
given the lack of available upper air observations at the time of its development, the NCM does
not describe the vertical structure of cyclones and the manner in which that vertical structure
supports the cyclone and evolves throughout its life cycle. In the remainder of this chapter we
will investigate (1) the nature of cyclogenesis (the intensification of a cyclone), (2) the process of
occlusion, and (3) the nature of cyclolysis (the decay of a cyclone).

9.3: Cyclogenesis

Now that we have provided an overview of QG theory, we are ready to consider the
physical sequence of events that characterize the adjustment of the mass and temperature fields
to a canonical cyclogenesis event. The original background state usually consists of a uniform,
westerly geostrophic flow. Along a uniform, westerly, geostrophic current, there is no advection
of geostrophic relative vorticity because there are no gradients in geostrophic vorticity.
Therefore, there can be no local changes in height owing to vorticity advection, and hence
vorticity advection alone cannot form an upper-level system in a uniform westerly current.

Nearly all cyclogenesis events proceed from a precursor upper-level disturbance in the
flow. This disturbance manifests itself as a relative vorticity maxima as illustrated in Figure
9.9(a). This can modeled conceptually as uniform westerly geostrophic flow upon which there is
superimposed a train of alternating cyclones and anticyclones; the result is wavetrain having
meridionally oriented troughs and ridges. There is no advection of geostrophic vorticity along the
trough and ridge axes, because the geostrophic vorticity there is a local maximum and minimum,
respectively. Therefore, heights cannot change locally along the ridge and trough axes owing to
vorticity advection, and hence vorticity advection there does not amplify the wave train.
However, due to vorticity advection, the disturbance will propagate in the direction of the flow.
Since the disturbance is often initially largest at middle and upper tropospheric levels, where the
geostrophic winds are often largest as well, there will be upward-increasing positive (negative)
vorticity advection (PVA (NVA)) downstream (upstream) of the disturbance. This circumstance
is associated with upward (downward) vertical motion downstream (upstream) of the trough axis
as shown in Figure 9.9(b).
Figure 9.9 Initial thermal and mass field adjustments to cyclogenesis. (a) Upper tropospheric vorticity maxima in a zonal thermal wind. Gray solid lines are 500 hPa geopotential heights, gray dashed lines are 500 hPa geostrophic absolute vorticity, and the black dashed lines are 1000-500 hPa thickness. “X” marks the location of the maximum absolute vorticity. (b) As for (a). Light (dark) shaded area is a region of upward (downward) vertical motion and upper tropospheric divergence (convergence). (c) As for (b) but for a subsequent time in the cyclone’s development. Note the development of the thermal ridge downstream of the upper trough axis and the thermal trough upstream of it.

However, differential temperature advection and differential diabatic heating can affect the height field so that upper-level systems form or intensify. Typically, an upper-level ridge forms or builds over a region of warm advection (i.e., temperature advection is decreasing with height and the height rises), and an upper level trough forms or deepens over a region of cold advection (i.e., temperature advection is increasing with height and the height falls). Under the influence of the cyclonic circulation associated with the developing lower tropospheric disturbance during the cyclogenesis process, low-level warm air advection will occur downstream of the upper-level trough axis and low-level cold air advection just upstream of it. As illustrated in Figure 9.10, such a circumstance will serve to raise the geopotential heights in the middle troposphere to the east of the surface low and lower the heights to its west.

Alternatively, the distribution of upper tropospheric convergence and divergence associated with the vertical motion couplet illustrated in Figure 9.10 will tend to increase the
upper tropospheric vorticity in the vicinity of the trough axis while decreasing it in the vicinity of the downstream ridge. A more intense upper-level vorticity maximum leads to greater PVA by the thermal wind and attendant upward vertical motions which further intensify the surface cyclone downstream of the upper feature. As the upper disturbance continues to develop and progress eastward, it begins to outrun its surface reflection. As a result, the convergence at the surface (maximized at the location of the sea-level pressure minimum) gradually becomes disconnected from its divergence valve aloft and the surface cyclone can no longer intensify. Thus, the phasing of the upper and lower disturbances is crucial to their complementary development.

**Figure 9.9 Schematic of temperature advection during cyclogenesis**

**9.3.1: Surface Cyclogenesis**

Based on the above discussion, it becomes important to discuss how the low-level cyclonic disturbance develops and intensifies. Intensification is often measured in terms of sea-level pressure decreases following the cyclone center. A consequence of this semi-Lagrangian negative pressure tendency is an increase in the low-level geostrophic vorticity. Thus, cyclogenesis can be viewed as a process of low-level vorticity production. Vorticity production necessitates the presence of divergence and vertical motions, as we have already seen from the QG vorticity equation. Consideration of the isobaric form of the continuity equation leads to

\[
\int_0^{p_s} d\omega = -\int_0^{p_s} (\nabla \cdot \mathbf{V}) dp \Rightarrow \omega(p_s) = -\int_0^{p_s} (\nabla \cdot \mathbf{V}) dp
\]
Now since
\[
\omega = \frac{Dp}{Dt} = \frac{\partial p}{\partial t} + \vec{V} \cdot \nabla p + w \frac{\partial p}{\partial z}
\]
and both \(w\) and \(\vec{V} \cdot \nabla p\) are nearly zero at the surface of the Earth, then we have
\[
\frac{\partial p_s}{\partial t} \approx - \int_0^{p_s} (\nabla \cdot \vec{V}) dp
\]
This expression, known as the **pressure tendency equation**, dictates that the surface pressure tendency at a point is a consequence of mass into the vertical column of the atmosphere above that point. Thus, net mass divergence (convergence) in the column is responsible for sea-level pressure falls (rises) at a given location. As we have already seen, however, measuring the divergence cannot be done with a great degree of accuracy. Thus approximation to the pressure tendency equation must be made in order to render useful result. The simplest approximation is through QG theory. The four QG forcing functions that are associated with rising motion and pressure falls at the surface are as follows:

- Vorticity advection becoming more cyclonic (or less anticyclonic) with respect to pressure
- A local maximum in temperature advection
- The vertical component of the curl of the frictional force become more cyclonic (or less anticyclonic) with respect to pressure
- A local maximum in diabatic heating

The first two forcing functions can be analyzed through the QG height tendency equation
\[
\left( \nabla^2 + \frac{f^2}{\sigma_p \partial p^2} \right) \chi = -f \vec{V}_g \cdot \nabla \left( \frac{1}{f} \nabla^2 \Phi \right) - \frac{f^2}{\sigma_p \partial p} \left[ \vec{V}_g \cdot \nabla \left( \frac{\partial \Phi}{\partial p} \right) \right]
\]
From QG theory, we find that surface height changes \(\chi\) are due to vorticity advection and divergence (or convergence) associated with vertical motions due to differential vorticity advection, temperature advection, differential friction, and diabatic heating.

Let us first consider the effects of differential vorticity advection alone. A typical wavetrain in the baroclinic westerlies aloft is depicted in Figure 9.10. There is positive vorticity advection (PVA) downstream from the maxima in absolute vorticity, which are located along the trough axes. Ordinarily, vorticity advection aloft is larger in magnitude than it is at the surface, where pressure (and height) systems tend to be circular. Therefore, vorticity advection is more cyclonic with height; therefore, there is rising motion downstream from upper-level troughs. It follows that the effect of differential vorticity advection is to make the surface pressure fall, and
hence contribute to the formation of a surface cyclone or trough. Similarly, downstream from a ridge aloft, the effect of differential vorticity advection is to make the surface pressure rise and contribute to the formation of a surface anticyclonic or ridge. Surface cyclones that form mainly in response to differential vorticity advection associated with a pre-existing finite-amplitude upper-level disturbance have been named Type B cyclones. Although they may be little surface temperature advection initially, it often becomes substantial as the cyclone deepens.

![Diagram](image)

Figure 9.10 The effects, according to quasigeostrophic theory, of differential geostrophic vorticity advection in a typical wavetrain in the baroclinic westerlies. Height contours at 500 mb in dam (solid lines); CVA (cyclonic vorticity advection) and AVA (anticyclonic vorticity advection) located downstream from vorticity maxima and minima, respectively, at 500 mb (top). Below 500 mb, vorticity advection is assumed to be relatively weak. Consequently, the vorticity advection is more cyclonic with height under the CVA region and more anticyclonic with height under the AVA region. The corresponding vertical velocity $\omega$ and horizontal divergence $\delta$ are depicted (bottom) in the following.

Let's consider the effects of temperature advection alone. Although surface cyclogenesis may occur along frontal zones, it does not happen in response to localized, intense, warm advection alone: If there were either a jet or a local concentration of temperature gradient, then
there would also be pre-existing differential vorticity advection. Bands of warm advection are often found along warm frontal zones. Since these bands represent relative maxima in temperature advection, there is a band of rising motion, which results in the intensification of the pre-existing frontal trough (Figure 9.11). Pressure ridges can intensify similarly in response to strong surface cold advection (Figure 9.11) behind cold frontal zones. The Polar-Front theorists expounded during the early part of this century on the importance of surface baroclinic zones in the formation of cyclones. Surface cyclones that form along frontal zones in the absence of a pre-existing finite amplitude upper-level disturbance are known as Type A cyclones.

![Figure 9.11 Illustration of temperature-advection patterns associated with surface fronts in a typical extratropical cyclone. QG theory argues that warm advection (WA) poleward of the warm front can form a trough at the surface, and cold advection (CA) to the rear of the cold front can form a ridge at the surface.](image)

It is this term that controls the development of disturbances from the perspective of the QG height tendency equation. We see that geostrophic temperature advection increasing (decreasing) upward is associated with height falls (rises). In a developing mid-latitude cyclone, middle tropospheric height rises (manifest as ridge building) typically occur to the east of the sea-level pressure (SLP) minimum, in the vicinity of the cyclone’s warm front. From the tendency equation perspective, this is a result of the fact that warm air advection is strong in the lower troposphere and weaker in the middle and upper troposphere in that region of the storm. The result is warm air advection decreasing with height (phenomenologically equivalent to cold air advection increasing with height) and height rises. To the west of the SLP minimum, in the vicinity of the surface cold front, middle tropospheric height falls occur consistent with the fact that the lower tropospheric cold air advection associated with the surface cold front is stronger than the middle and upper tropospheric cold air advection in the same location. Thus, the
geostrophic temperature advection increases with height in that vicinity and, consequently, mid-tropospheric height falls occur there.

The height falls to the west of the developing SLP minimum lead to a positive, mid-tropospheric geostrophic vorticity tendency there, while the height rises to the east of the SLP minimum are associated with a negative, mid-tropospheric geostrophic vorticity tendency there, as illustrated in (Figure 9.12). The juxtaposition of positive and negative geostrophic vorticity tendencies, in turn, promotes more intense positive vorticity advection by the thermal wind (and consequent upward vertical motion) in the vicinity of the SLP minimum leading to continued lower tropospheric cyclogenesis. In this way, the asymmetric temperature advection field associated with a developing mid-latitude cyclone makes a significant contribution to the dynamics of cyclogenesis. Of course, nature is nearly always more complicated than the simplest example and careful analysis of the geostrophic temperature advection profile is necessary to employ the QG tendency diagnostic usefully in any given real case.

Figure 9.12 The effect of horizontal temperature advection on geopotential tendency. Solid black arrows are streamlines of the lower tropospheric thermal wind. Surface low-pressure center is indicated with $L$ and the gray arrows represent the lower tropospheric winds associated with the storm. Light (dark) shaded region identifies an area where warm (cold) air advection decreases with height leading to height rises (falls) and a negative (positive) vorticity tendency.
The effects of differential friction alone are now considered. In a cyclone whose geostrophic vorticity is independent of height in the friction layer, the vertical component of the curl of the frictional force is anticyclonic and becomes zero at the top of the friction layer (Figure 9.13). There is therefore, rising motion in and just above the friction layer (called **Ekman pumping**), while in the friction layer there is convergence; the effect of convergence is to intensify the cyclone, while the direct effect of friction is to dissipate energy and weaken the cyclone. However, just above the top of the friction layer there is divergence and no friction, and hence the cyclone weakens: The effects of friction have been communicated to the free atmosphere by the secondary circulation. In the region of a surface anticyclone, there is similarly sinking motion in the friction layer only, while at the surface, there is divergence; just above the friction layer there is convergence, and the anticyclone weakens.

Diabatic heating alone may be responsible for the formation of a surface cyclone, if the region of diabatic heating represents a local maximum. For example, intense localized diabatic heating owing to latent heat release from cumulus convection that acts for a time long enough so that the Earth's rotation is dynamically important may form a low or a trough. The upward flux of heat and water vapor from the ocean surface and the vertical distribution of latent heat release through cumulus convection is thought to be responsible for tropical cyclogenesis. It is difficult, however, to model the effect of small-scale cumulus convection on the larger scale in a realistic
manner. On the other hand, a region of diabatic cooling owing to evaporation of rain in an unsaturated layer below may be responsible for the formation of a high or a ridge.

### 9.3.2: Explosive Cyclogenesis

The foregoing description of cyclogenesis makes no mention of the fact that clouds and precipitation (and therefore latent heat release) are involved in this process. Naturally, the interaction between the dynamic and diabatic processes is an important aspect of the overall process of cyclogenesis. Though such interaction characterizes every cyclogenesis event to some degree, these interactions are most vividly illustrated by considering cases of dramatic surface development, known as **explosive cyclogenesis**.

![Figure 9.14 Distribution of 24 h deepening rates for all northern hemisphere surface cyclones in year. The dark solid line indicates the sum of two normal curves while the gray lines and shadings represent the separate distributions (light shading for the 'ordinary' and darker for 'explosive' cyclones). Adapted from Roebber (1984)](image)

Explosive cyclogenesis is the rapid development of a sea-level pressure minimum. Prior work has suggested a threshold of 24 hPa of deepening in 24 hours as a reasonable distinguishing characteristic of an explosive deepener. The deepening rates for all northern hemisphere cyclones in a single year are shown in Figure 9.14. It appears that the distribution is skewed toward these rapid deepeners suggesting that something may be different about these storms. In fact, there are some notable differences between the ‘ordinary’ cyclones that constitute the
majority of all cyclones and these rarer events. One of the more significant differences between these populations is that the explosive deepeners not only deepen more rapidly but also for a longer time than the ‘ordinary’ cyclones.

What does this mean about the contrast in physical processes that operate in these two populations? The distribution of these explosive deepeners provides a clue as to the circumstances that conspire to produce them. Modern research suggests that explosively deepening mid-latitude cyclones in the northern hemisphere tend to develop along the warm western boundary ocean currents such as the Kurishio and Gulf Stream. The prevailing view is that these storms are the manifestation of physical and dynamical processes that occur to some degree in all cyclones but which are particularly vigorous in explosive deepeners.

A reasonable next question, then, is: “What makes ordinary processes so potent in these storms?” Recalling that surface development is strongly tied to upward vertical motion through the vorticity equation, this suggests that surface development is forced by thermal advection and vorticity advection. As we will show next chapter, the local static stability of the atmosphere acts as the amplitude modulator for the vertical velocity. Hence, the prevalence of explosively deepening cyclones over warm ocean currents is a result of the fact that these locations are characterized by consistently lower static stability and, consequently, a consistently more vigorous response to forcing for upward vertical motion. These more intense vertical motions then lead to more intense cyclogenesis. But even this physical linkage does not yet reference the effect of the characteristic cloud and precipitation distribution of cyclones.

The characteristic precipitation distribution associated with mid-latitude cyclones is asymmetric. The period of most rapid development occurs when heavy precipitation develops poleward and westward of the cyclone center. The associated latent heat release (LHR) can (1) add energy to the system, (2) focus and intensify the vertical motion pattern through a local reduction of the static stability in saturated updrafts, and, perhaps most interestingly, (3) affect the structure and dynamics of the larger-than-cyclone scale so as to intensify the cyclogenetic effect of ordinary dynamical processes. This last point bears special attention as it lies at the heart of a conceptual/dynamical model of cyclogenesis known as the self-development paradigm.

Consider, as a first example, the feedback from sensible and latent heat fluxes in the lower troposphere to cyclogenesis. As shown schematically in Figure 9.15, poleward-directed boundary layer winds coupled with ascent on the eastern side of the cyclone warm the lower troposphere on the equatorward side of the developing warm front via sensible heating associated with the warm advection and diabatic heating resulting from LHR in the moist, ascending air. This warming leads to an increase in the magnitude of the low-level temperature gradient and a consequent increase in the magnitude of the warm advection there. Stronger warm advection is often associated with intensified ascent in that location. Greater ascent leads to more
intense baroclinic energy conversion and often to a stronger cyclone whose intensified circulation, in turn, results in a positive feedback loop.

Figure 9.15 Schematic illustration of the influence that sensible and latent heat fluxes in the planetary boundary layer can have on the magnitude of the lower tropospheric temperature advection east of the surface cyclone center. (a) Prior to the influence of the heat fluxes a uniform temperature gradient exists. Dashed lines are isotherms, ‘L’ is the location of the sea-level pressure minimum, arrows represent the flow around the cyclone and the gray shaded area is the location where heat fluxes will warm the boundary layer. (b) Increased temperature gradient results from the heating in the boundary layer. Intensified lower tropospheric warm air advection intensifies the cyclone. (c) More intense cyclone leads to more intense lower tropospheric winds (bolder arrows) and increased warm air advection.

On larger scales, the latent heat release associated with the enhanced cloud and precipitation production produces a positive thickness anomaly just east of the upper-level shortwave trough axis. Consequently, the geopotential heights in the middle and upper troposphere increase in that region and a small-scale ridge is built up above the latent heating maxima. Positive vorticity advection to the east of the upper-level short-wave compels that feature to move eastward. In the face of the diabatic ridge building that occurs in association with the latent heat release in the cloud shield to the east, the wavelength between the upstream trough and downstream ridge axes shrinks. As a result of this wavelength shortening, the magnitude of the cyclonic vorticity advection by the thermal wind downstream of the trough axis greatly intensifies leading to more intense upward vertical motions. The stronger vertical motions intensify the cyclogenesis and produce more latent heat release just downstream of the upper-level short-wave which tends to further shorten the wavelength of the upper disturbance. In this way, a positive feedback loop is established.

A large number of numerical modeling studies investigating the influence of latent heat release on cyclogenesis have been undertaken in the past 30 years. The consensus conclusion drawn from these studies is that since water vapor is not a passive scalar, its phase change tends to concentrate normal baroclinic processes onto smaller scales, which leads to feedbacks that further the scale contraction and intensification of these explosively deepening storms. From that perspective, it becomes clear that these storms do not arise as a consequence of ‘special’ dynamical processes but rather as a result of uncommonly intense interactions among the
‘ordinary’ suite of physical and dynamical processes that operate, to some degree, in all mid-latitude cyclones.

9.3.3: Cyclogenesis and Frontogenesis

Recall that the NCM suggested that cyclones form along a pre-existing polar front that divided polar from tropical air masses throughout the depth of the troposphere. Subsequent work has proven beyond a doubt that cyclogenesis and frontogenesis are nearly concurrent processes. An idealized wave train superimposed upon a zonally oriented baroclinic zone is illustrated schematically in Figure 9.16. As the perturbations develop, regions of deformation develop as indicated. The meridional shear induced by the disturbances produces thermal ridges and troughs while the deformation (specifically diffluence) to the northeast and southwest of each cyclonic disturbance in the wave train provides an environment in which the gradient of any variable in the flow may be intensified. This applies, of course, to temperature and consequently these two regions of deformation act frontogenetically to produce the warm and cold frontal zones. This
simple idealized illustration demonstrates that the development of fronts is a consequence, not a cause, of cyclogenesis – a conclusion that departs radically from the ideas put forth in the NCM.

In defense of the NCM, baroclinic instability theory indicates that a substantial background vertical shear, made manifest in a robust horizontal temperature contrast (through thermal wind balance), is necessary in order for cyclogenesis to occur. This temperature contrast, coupled with the presence of a discernible upper tropospheric short-wave trough, provides an environment in which cyclonic vorticity advection by the thermal wind can initiate the upward vertical motions necessary to spin-up the low-level cyclone. The lifting occurs simultaneously with the production of a thermal ridge displaced slightly downstream of the developing low-level circulation center. The circulation itself then deforms the background baroclinic zone via differential horizontal advections. Such differential horizontal advections produce regions of intensified baroclinicity (via frontogenesis) and quasi-linear vertical motion couplets which, when added to the cellular elements contributed by the shearwise forcing, result in the characteristic comma-shaped pattern of vertical motion associated with the mid-latitude cyclone. Note that this view also presents a life cycle in which the cyclogenesis and frontogenesis are nearly concurrent processes, perhaps even with cyclogenesis somewhat leading frontogenesis.

In the last few decades, a conveyor belt model of cyclone structure and evolution has emerged, as shown in Figure 9.17. The model consists of: (1) a warm conveyor belt, which transports warm air poleward; (2) a cold conveyor belt, which runs parallel to the warm front on its cold side; and (3) a dry descending air stream behind the surface cold front. In this model, the orientation of the warm conveyor belt largely determines the distribution of precipitation for the cyclone. The forward-sloping warm conveyor belt, as shown in Figure 9.18(a), is similar in many ways to a cold katafront. A forward-sloping warm conveyor tends to confine clouds and precipitation along and ahead of the surface cold front in the warm sector. This type of system commonly occurs downstream of a diffluent trough axis in the flow aloft. The rearward-sloping warm conveyor belt, as shown in Figure 9.18(b), is similar to a cold anafront. This leads to heavy precipitation at the time of frontal passage, followed by lighter, more stratiform precipitation behind the surface front. This type of system commonly occurs downstream of a confluent trough axis in the flow aloft.
Figure 9.17 Schematic of the conveyor belt model of cyclogenesis

Figure 9.18 (a) Schematic of the forward-sloping conveyor belt. (b) Schematic of rearward-sloping conveyor belt

Thus far we have only considered the dynamics of the cyclogenesis phase of the mid-latitude cyclone life cycle. We will now look at the movement of a mature mid-latitude cyclone. The NCM introduced the notion of occlusion as the peak of intensity and commencement of
decay. In the next section we will discuss the essential distinguishing characteristics of the occluded phase of the life cycle as well as the characteristic dynamics that operate in that phase.

9.4: The Post-Mature Stage

Since the notion of occlusion was first introduced, considerable controversy has existed concerning the nature of the occluded (post-mature) stage of the mid-latitude cyclone life cycle. Surprisingly, much of this controversy has centered around the means by which the characteristic occluded thermal structure evolves in the post-mature cyclone. In this section we therefore examine (1) the characteristic occluded thermal structure itself, and (2) an underlying dynamical mechanism (diagnosed using QG theory) that simultaneously accounts for the development of that thermal structure and for the characteristic presence of ascent in the occluded quadrant of mid-latitude cyclones. We begin by briefly reviewing aspects of the occluded thermal structure.

As far back as the 1920s it was suggested that the process of occlusion involved the cold front encroaching upon, and subsequently overtaking and ascending, the warm frontal surface. One of the main results of this process of warm occlusion was the production of a wedge of warm air aloft, displaced poleward of the surface warm and (newly created) occluded fronts. The cloudiness and precipitation associated with the development of the warm occlusion were suggested to result from lifting of warm air ahead of the upper cold front and were consequently distributed to the north and west of the sea-level pressure minimum. As a result of the gradual squeezing of warm air aloft between the two intersecting frontal surfaces, the horizontal thermal structure of a warm occlusion was characterized by a thermal ridge connecting the peak of the warm sector to the geopotential or sea-level pressure minimum. This thermal ridge is often manifested as a 1000–500 hPa thickness ridge or an axis of maximum $\theta$ in a horizontal cross-section as shown in Figure 9.19. Note that considerable upward vertical motion is also co-located with this thermal ridge.
Figure 9.19 The characteristic occluded thermal ridge as observed at 0600 UTC 1 April 1997. (a) Solid lines are 1000–500 hPa thickness labeled in dam and contoured every 6 dam. Dashed lines with shading are 700 hPa upward vertical motion labeled in cm/s and contoured every 5 cm/s. Both variables are from an 18 h forecast of the NCEP Eta model valid at 0600 UTC 1 April 1997. (b) Solid lines are 18 h forecast of 700 hPa $\theta$, valid at 0600 UTC 1 April, labeled in K and contoured every 2 K. Vertical motion as in (c). As for (b) but solid lines are 18 h forecast of 700 hPa $\theta$, labeled in K and contoured every 4 K. Vertical motions indicated as in (a). Vertical cross-sections along lines B – B’ and C – C’ are shown in Figure 9.20.
Figure 9.20 (a) Vertical cross-section of $\theta$, through the occluded thermal ridge, along line B – B’ in Figure 9.19(c). Solid lines are isentropes labeled in K and contoured every 3 K. Shaded regions are upward vertical motions labeled in cm/s and contoured every 5 cm/s. ‘A’ represents the point of intersection between the cold and warm frontal zones in the warm occluded thermal structure. (b) As for (a) but for the cross-section along line C – C’ in Figure 9.19(c).

A vertical cross-section perpendicular to the axis of the thermal ridge in Figure 9.19 reveals the characteristic vertical structure of the warm occlusion (Figure 9.20a) consisting of a poleward-sloping axis of maximum $\theta$ separating two regions of concentrated baroclinicity. The surface warm occluded front is generally analyzed at the location where this axis of maximum $\theta$ intersects the ground, whereas the base of the warm air between the two baroclinic zones (the cold and warm fronts) sits atop their point of intersection (labeled A in Figure 9.20a). It is clear, however, from this cross-section that the upward vertical motion maximum is located at the leading edge of the cold frontal baroclinicity, significantly displaced from the position of the surface occluded front. Upon taking another vertical cross-section further along the thermal ridge.
toward the surface cyclone center (Figure 9.20b), we find that the same basic thermal structure and a roughly similar vertical motion distribution exist though the intersection of the warm and cold frontal baroclinic zones (labeled A in Figure 9.20b also) occurs at a higher elevation.

The observation that the cloudiness and precipitation characteristic of the occluded quadrant of cyclones often occurs in the vicinity of the thermal ridge led scientists at the Canadian Meteorological Service in the 1950s and 1960s to regard the essential feature of a warm occlusion to be the trough of warm air that is lifted aloft ahead of the upper cold front, not the position of the surface occluded front. The sloping line of intersection between the cold and warm frontal baroclinic zones, termed the trough of warm air aloft (also called trowal), was found to bear a closer correspondence to the cloud and precipitation features in occluded North American cyclones than did the often weak surface warm occluded front. The trowal marks the 3-D sloping intersection of the upper cold frontal portion of the warm occlusion with the warm frontal zone and therefore represents a refined, 3-D description of the warm occluded structure presented in the NCM. A schematic illustrating the trowal conceptual model is shown in Figure 9.21.

Figure 9.21 Schematic of the trowal conceptual model. The dark (light) shaded surface represents the warm edge of the cold (warm) frontal zone. The bold dashed line at the 3-D sloping intersection of those two frontal zones lies at the base of the trough of warm air aloft – the trowal. The schematic precipitation in the occluded quadrant of the cyclone lies closer to the projection of the trowal to the surface than to the position of the surface warm occluded front.

Given the availability of gridded output from numerical simulations of cyclones along with the graphical capability of software display packages for viewing this output, it is now
relatively simple to identify the trowal structure in occluded cyclones. Referring back to Figure 9.20, notice that the 312 K isentrope lies near the warm edge of both the warm and cold frontal baroclinic zones comprising the warm occluded structure. Plotting the 312 K moist isentrope every 100 hPa beginning at 1000 hPa from a gridded data set of this case reveals the isobaric topography of the 312 K surface (Figure 9.22a). Clearly identifiable in this topography are (1) the steeply sloped cold frontal surface, (2) the less steeply sloped warm frontal surface, and (3) the poleward- and westward-sloping 3-D ‘canyon’ in the 312 K surface representing the trowal. This topography can also be viewed through inspection of the actual 312 K surface produced by a different software package (Figure 9.22b).

Despite the historical controversy surrounding the nature of the occlusion process, there is fairly widespread agreement that the thermal structures just described are among the basic structural characteristics of the post-mature phase mid-latitude cyclone.
9.5: The Decay Stage

The decay stage of the extratropical cyclone is the least studied, and therefore the least well-understood stage of the cyclone life cycle. At a basic level, the decay stage is associated with lower tropospheric geopotential and sea-level pressure rises and, consequently, with a systematic decrease in lower tropospheric vorticity. Therefore, cyclone decay is known as cyclolysis – the opposite of cyclogenesis. We have already seen that cyclogenesis requires column stretching and upward vertical motions. It therefore seems reasonable to assume that cyclolysis requires column squashing and downward vertical motions. Certainly this set of physical circumstances represents a sufficient means of reducing the lower tropospheric vorticity; it does not, however, appear to be necessary for the occurrence of surface cyclolysis.

Recall that the vertical structure of a developing mid-latitude cyclone was such that the axis of minimum geopotential height tilted into the vertical shear (i.e. to the west as shown in Figure 9.5b). Since upward vertical motions occur downshear of upper-level vorticity maxima (i.e. minima in the upper-level geopotential height), that vertical structure ensures that an upper-level, dynamically forced divergence maximum is located directly above the sea-level pressure minimum and, via the resulting upward vertical motions, serves to evacuate the mass that accumulates into the sea-level pressure minimum as a consequence of frictionally induced surface convergence. As the cyclone matures, the vertical tilt of the geopotential minimum axis gradually becomes more vertical by the time of occlusion. A purely vertical stacking results in the displacement of the upper divergence maximum to the east of the sea-level pressure minimum.

By the commencement of decay, the sea-level pressure minimum has reached its greatest intensity as has the frictionally induced surface convergence into its center. As a consequence of the eastward displacement of the upper-level divergence at this stage of the life cycle, there is no mechanism available to evacuate the accumulating mass near the center of the surface cyclone and the surface pressure rises as a consequence. This rise in surface pressure is associated with a decrease in the near surface geostrophic vorticity and therefore qualifies as a cyclolysis event. Note that this sequence of events can occur in the absence of any notable downward vertical motions over the surface cyclone center. Instead, it is the absence of upward vertical motions sufficient to evacuate the mass accumulated near the center of the surface cyclone that appears to be the dynamically necessary ingredient for cyclone decay. Indeed, any process that results in decreased upper-level divergence directly above the surface cyclone center leads to cyclone decay.
The results of a recently constructed synoptic climatology of surface cyclolysis in the north Pacific Ocean can be used to illustrate these characteristic elements of the decay stage. In particular, we examine the composite evolutions of the 500 hPa geopotential height and sea-level pressure distributions constructed from 180 so-called rapid cyclolysis periods (RCPs), defined as 12 h periods during which a sea-level pressure rise of at least 12 hPa occurs at the center of a mid-latitude cyclone. Twenty-four hours before the commencement of rapid cyclolysis, a fairly intense sea-level pressure minimum is located just downstream of a strongly curved, slightly negatively tilted, 500 hPa geopotential height trough (Figure 9.23a). Twelve hours later, the upper trough axis has become more negatively tilted and the more intense sea-level pressure minimum has drawn closer to the trough axis, characteristic of occluded cyclones (Figure 9.23b).
By commencement of the 12h period of rapid cyclolysis (Figure 9.23c), the sea-level pressure minimum lies directly beneath the 500 hPa geopotential height minimum of the even more negatively tilted trough. An astounding transformation in the 500 hPa geopotential height field occurs during the 12 h RCP. The radius of curvature of the geostrophic streamlines increases dramatically while the 500 hPa geopotential height gradient weakens south of the dramatically weaker sea-level pressure minimum (Figure 9.23d). The rapid flattening of the 500 hPa trough–ridge couplet, which had been amplifying up to the commencement of cyclone decay, is associated with a rapid decrease in upper tropospheric divergence downstream of the upper trough axis (i.e. to the northeast of the sea-level pressure minimum). Such a circumstance, occurring immediately after the surface cyclone reaches its maximum intensity, provides the key ingredient for the subsequent rapid cyclolysis at the surface.

As the surface cyclone reaches its greatest intensity, presumably so does the lower tropospheric mass convergence into it, forced by friction in the lower troposphere. With the abrupt reduction in cyclonic curvature and, consequently, in the mass divergence aloft, the accumulating mass in the lower troposphere is less efficiently evacuated from the column and the sea-level pressure rises rapidly as a result. Through subsequent study of these events, it appears that rapid surface cyclolysis, though influenced by friction in the boundary layer, is initiated and largely controlled by synoptic-scale dynamical processes. Less intense ‘garden variety’ cyclolysis events most likely proceed in a similar fashion relying more on the gradual acquisition of a downshear tilted structure, characteristic of all cyclones in the post-mature phase, than on the eradication of upper tropospheric flow curvature.
Chapter 10: Introduction to Convective Initiation

The previous chapters focused on the dynamics of synoptic-scale motions circulations. Such large-scale motions are strongly influenced by the rotation of the earth so that the Coriolis force dominates over inertial forces. To a first approximation, as shown in Chapter 6, large-scale motions can be modeled by quasi-geostrophic theory. The study of quasi-geostrophic motions has been a central theme of dynamic meteorology for many years. Not all important circulations fit into the quasigeostrophic classification, however. Such circulations include a wide variety of phenomena. They all, however, are characterized by horizontal scales that are smaller than the synoptic scale (i.e., the macroscale of motion), but larger than the scale of an individual fair weather cumulus cloud (i.e., the microscale). Hence, they can be classified conveniently as mesoscale circulations. Most severe weather is associated with mesoscale dynamics. Thus, understanding of the mesoscale is of both scientific and practical importance.

Mesoscale dynamics is generally defined to include the study of motion systems that have horizontal scales in the range of about 10 to 1000 km. It includes circulations ranging from thunderstorms and internal gravity waves at the small end of the scale to fronts and hurricanes at the large end. Given the diverse nature of mesoscale systems, it is not surprising that there is no single conceptual framework, equivalent to the quasi-geostrophic theory, that can provide a unified model for the dynamics of the mesoscale. Indeed, the dominant dynamical processes vary enormously depending on the type of mesoscale circulation system involved.

Possible sources of mesoscale disturbances include instabilities that occur intrinsically on the mesoscale, forcing by mesoscale thermal or topographic sources, nonlinear transfer of energy from either macroscale or microscale motions, and interaction of cloud physical and dynamical processes. Energy transfer from small scales to the mesoscale is a primary energy source for mesoscale convective systems. These may start as individual convective cells, which grow and combine to form thunderstorms, convective complexes such as squall lines and even hurricanes. Conversely, energy transfer from the large scale associated with temperature and vorticity advection in synoptic-scale circulations is responsible for the development of frontal circulations. Thus, an understanding of convective initiation is vital in understanding the development of mesoscale systems. In this chapter, we will primarily examine the two major types of fluid instabilities that lead to convection and mesoscale phenomena: gravitational instability and convective instability. We first start with a definition of lapse rate.
10.1: Lapse Rates

Before we discuss any elements of mesoscale convection, we need to give a more precise definition of an air parcel. We are going to define assume that an air parcel is an infinitely small parcel air that has the following properties:

1. It is thermal insulated from its environment so that heat is not added to or taken away from the parcel (i.e. the parcel’s environment is adiabatic).
2. It remains at exactly the same pressure as the environmental air at the level. In other words, the parcel immediately adjusts to the hydrostatic pressure at that level and does not disturb the surrounding air when it is vertically displaced.
3. It moves slowly enough so that its macroscopic kinetic energy is small compared to the kinetic energy of the surrounding air.

Although this definition has some limitations, this definition allows us to understand some basic features about rising and sinking air masses.

Recall from Chapter 3 that the first law can be used to explain why rising air tends to cool whereas sinking air tends to warm. Suppose that we have a rising air parcel that is insulated from its environment such that no heat is added or taken away from the parcel. This is known as an adiabatic process. As the air parcel rises, it enters into regions of lower pressure. This causes the
air parcel to expand since the lower pressure outside allows the air molecules to push out on the parcel walls. This means that air parcels is doing work on the environment at the expense of its own internal energy. Therefore, the parcel will cool as it rises.

How does the temperature change with height? The answer to this question deals with the definitions of lapse rates. Lapse rate is defined as the decrease of temperature with height. It can be shown that there are four different lapse rates that are relevant to our discussion:

1. **Environmental lapse rate** ($\Gamma$): The rate at which temperature decreases with height in the environmental air. This is measured using a radiosonde and varies from place to place and from time to time.
2. **Dry adiabatic lapse rate** ($\Gamma_d$): The rate at which temperature decreases with height in a dry air parcel. It can be shown that this is $9.8 \, °C/km$.
3. **Saturated adiabatic lapse rate** ($\Gamma_s$): The rate at which temperature decreases with height in a saturated air parcel. This varies depending upon the moisture in the parcel, but values of $\Gamma_s$ range from $4 \, °C/km$ near the ground in warm, humid air to $6 – 7 \, °C/km$ in the middle troposphere.
4. **Dew point lapse rate** ($\Gamma_{dew}$): The rate at which the dew point temperature decreases with height in an unsaturated parcel. Here, the moisture content of the parcel stays the same, but the pressure of the parcel varies. Values of $\Gamma_{dew}$ range from $1.6 \, °C/km$ to $2.0\, °C/km$. Once a parcel is saturated, then $\Gamma_{dew} = \Gamma_s$.

**10.1.1: Derivation of $\Gamma_d$ and $\Gamma_s$**

Recall that the thermodynamic energy equation can be written in terms of potential temperature

$$\frac{d\theta}{dt} = \frac{\theta \dot{Q}}{c_p T}$$

For an adiabatic process, we have

$$\frac{d\theta}{dt} = 0$$

To determine $\Gamma_d$, we take the logarithm and differentiating with respect to height
Using the ideal gas law and the hydrostatic equation, we obtain

\[
\frac{1}{\theta} \frac{d\theta}{dz} = \frac{1}{T} \frac{dT}{dz} - \frac{R}{P_c} \frac{dP}{dz}
\]

Since \( \theta \) is constant with height then

\[-\frac{dT}{dz} = \frac{g}{c_p} = \Gamma_d\]

For dry air, \( \Gamma_d = g/c_p = 9.8 \, ^\circ C/km \approx 1^\circ C/100 \, m \). Note that the dry adiabatic lapse rate is independent of temperature and pressure. Hence, \( \Gamma_d \) gives the rate of change of temperature of a dry air parcel that is being raised (cooling) or lowered (warming) adiabatically in the atmosphere.

As an air parcel is lifted it cools dry adiabatically until it becomes saturated. Further ascent result in condensation. This releases latent heat and the parcel cools at a lesser rate than the dry adiabatic lapse rate. To determine \( \Gamma_s \), we start with the first law of thermodynamics

\[dU = dQ + dW\]

For a unit mass of gas (which is appropriate for an air parcel), we have \( dW = -P \, d\alpha \), where \( \alpha \) is the specific volume. Therefore, we have

\[dU = dQ - P \, d\alpha\]

Based on the result of Joule’s experiment, \( dU = c_v dT \) and thus, we have

\[dQ = c_v dT + P \, d\alpha = c_v dT + d(P\alpha) - \alpha dP = c_v dT + R \, dT - \alpha dP\]

If the pressure is constant, then

\[dQ = (c_v + R) dT \Rightarrow \left(\frac{dQ}{dT}\right)_p = c_v + R = c_p\]

By substitution, we have \( dQ = c_p dT - \alpha dP \). Using the hydrostatic equation, we have

\[dQ = c_p dT + g \, dz\]
If the saturation mixing ratio of the air with respect to water is \( w_s \), the quantity of heat released into (or absorbed from) a unit mass of dry air due to condensation (or evaporation) of liquid water is \(-L_v \, dw_s\), where \( L_v \) is the latent heat of condensation. Therefore, we have

\[-L_v \, dw_s = c_p \, dT + g \, dz\]

Dividing by \( c_p \, dz \) and rearranging gives

\[
\frac{dT}{dz} = -\frac{L_v \, dw_s}{c_p \, dz} - \frac{g}{c_p} = -\frac{L_v \, dw_s}{c_p \, dT} \, dz = -\frac{dT}{dz} \left[ \frac{L_v \, dw_s}{c_p \, dT} + \frac{g/c_p}{dz} \right]
\]

where the second equality uses the chain rule. Rearranging this expression gives

\[
1 + \frac{L_v \, dw_s}{c_p \, dT} = \frac{g/c_p}{dT/dz} \Rightarrow \frac{dT}{dz} = \frac{g/c_p}{1 + \frac{L_v \, dw_s}{c_p \, dT}}
\]

Therefore, the saturated adiabatic lapse rate is given by

\[
\Gamma_s = -\frac{dT}{dz} = \frac{\Gamma_d}{1 + \frac{L_v \, dw_s}{c_p \, dT}}
\]

Since \( dw_s/dT \) is always positive, then \( \Gamma_s < \Gamma_d \), emphasizing the point that the release of latent heat counterbalances the adiabatic cooling of a rising air parcel.

### 10.2: Static Stability

Lapse rates are typically used to assess the static stability in the atmosphere. Static stability is defined as the ability of a fluid at rest to become turbulent or laminar due to the effects of buoyancy alone. A fluid is considered statically unstable when it becomes turbulent, is considered statically stable when it becomes laminar, and statically neutral when it remains laminar or turbulent depending on its history.
Let’s consider the condition for $\Gamma < \Gamma_s$ as shown in Figure 10.2. If a parcel is forced to rise to the level of points $A$ and $B$, then the parcel temperature $T_A$ is lower than the environmental temperature $T_B$. As the parcel’s pressure immediately adjust to the pressure of the environment, then the ideal gas equation implies that the density of the parcel is greater than that of the environmental air. Thus, the parcel will therefore sink down back to its original level. Conversely, if the parcel is displaced downward from $O$, it becomes warmer than the ambient air and if left to itself will tend to rise back to $O$. In both cases, the air parcel encounters a restoring force after being displaced, which inhibits vertical mixing (the greater the difference $\Gamma_s - \Gamma$, the greater the restoring force and the greater the static stability. Therefore, $\Gamma < \Gamma_s$ corresponds to an **absolutely stable stratification**. Absolutely stable layers in the atmosphere can form by radiative cooling at the surface, cold air moving at low-levels, and/or warm air moving over cold ground.

Let’s consider the condition for $\Gamma > \Gamma_d$ as shown in Figure 10.3. If a parcel is forced to rise to the level of points $A$ and $B$, then the parcel temperature $T_A$ is greater than the environmental temperature $T_B$. As the parcel’s pressure immediately adjust to the pressure of the environment, then the ideal gas equation implies that the density of the parcel is less than that of the environmental air. Thus, the parcel will therefore continue to rise. Similarly, if the parcel is displaced downward it will be cooler than the ambient air and will continue to sink if left to itself. In both cases, the air parcel accelerates away from equilibrium (the greater the difference $\Gamma_d - \Gamma$, the greater the acceleration). Therefore, $\Gamma > \Gamma_d$ corresponds to an **absolutely unstable stratification**. Such unstable conditions generally do not exist in the atmosphere for long because this instability is reduced by strong vertical mixing. An exception to this is in the layer just above the ground when there is strong heating from below. Absolutely unstable layers can occur when cold air is moving aloft (this often occurs when an extra-tropical cyclone passes over
head), surface heating (this suggests that the atmosphere is most unstable in mid-afternoon), warm air moving in at low level (this often occurs ahead of a cold front), and/or cold air moving over a warm surface (an example of this is lake-effect snow).

Let’s consider the condition for $\Gamma_s > \Gamma > \Gamma_d$ as shown in Figure 10.4. If a parcel is lifted from its equilibrium level $O$, it will cool adiabatically until it becomes saturated at point $A$. This location is called the lifting condensation level (LCL). During lifting, the mixing ratio $q$ and the potential temperature $\theta$ of the air parcel remain constant, but the saturation mixing ratio $q_s$
decreases until it becomes equal to $q$ at the LCL. In other words, at the LCL, $T = T_d$. The LCL is used to estimate boundary layer cloud base heights. At the LCL, the parcel is colder and denser than the ambient air.

Further lifting will cause the parcel to cool at the saturated adiabatic lapse rate. If the parcel is sufficiently moist, the moist adiabat (i.e. lines of $\Gamma_s$) through A will cross the environmental temperature sounding at B. Up to point B, the energy is required to force the parcel above its equilibrium level O. Above B, the parcel is now warmer than the environment and this positive buoyancy results in the upward motion of the parcel even without the forced lifting. The location B is called the level of free convection (LFC) – the LFC is dependent on the amount of moisture in the parcel as well as the magnitude of the environmental lapse rate $\Gamma$. When $\Gamma_d > \Gamma > \Gamma_s$, the atmosphere is said to be conditionally unstable. As mentioned above, note that an unsaturated (saturated) parcel will be cooler (warmer) than the environment. This implies that the unsaturated (saturated) parcel will sink (rise). Hence, for conditional instability, the parcel is unstable if it's saturated.

We can summarize our discussion of a rising air parcel. As an air parcel rises in the atmosphere, it adiabatically cools (i.e. the air parcel rises according to the dry adiabatic lapse rate $\Gamma_d$) until it becomes saturated. The height at which this occurs is called the lifting condensation level (LCL). If the air parcel is lifted further beyond the LCL, water vapor in the air parcel will begin condensing, forming cloud droplets. The LCL is a good approximation of the height of the cloud base which will be observed on days when air is lifted mechanically (e.g. due to surface convergence or frontal lifting) from the surface to the cloud base. Moreover, if the air parcel is lifted beyond the LCL, it will rise at the saturated adiabatic lapse rate $\Gamma_s$. If it continues to rise, it may reach a height at which the parcel's temperature is warmer than the environmental temperature. In other words, the environmental lapse rate is less than the moist adiabatic lapse rate. At this level, mechanical forcing is no longer needed to lift the parcel because the parcel is positively buoyant with respect to its environment. This level is called the level of free convection. As the parcel continues to rise beyond the LFC due to its own buoyancy, it will eventually reach a height where the buoyant lifted parcel becomes neutrally buoyant. This height is defined as the equilibrium level (EL). This is the height above the LFC at which the parcel temperature is equal to the environmental temperature.

**10.2.1: Mathematical Treatment of Static Stability**

As shown in the previous section, in a statically unstable (stable) environment, a vertically displaced air parcel is accelerated away from (returned to) its equilibrium position. The buoyant force is what causes the static instability of air parcels and drives the vertical circulation. The buoyancy force is a vertical pressure gradient force that is not balanced with gravity and is attributable to variations in density within a column. The dynamics of many mesoscale
phenomena are often more intuitive if the vertical momentum equation is written in terms of a buoyant force. We can write the frictionless vertical equation of motion as

\[
\frac{dw}{dt} = -\frac{1}{\rho} \frac{dp}{dz} - g
\]

Thus, vertical motion is due to the imbalance between the pressure gradient force and the acceleration due to gravity. Now if we assume that the environmental air is in hydrostatic balance, then

\[
\frac{dp}{dz}_{env} = -\rho_{env}g
\]

However, for an air parcel,

\[
\frac{dw}{dt} = -\frac{1}{\rho_{par}} \left( \frac{dp}{dz} \right)_{par} - g
\]

Using the assumptions of parcel theory, we assume that

\[
\left( \frac{dp}{dz} \right)_{par} = \left( \frac{dp}{dz} \right)_{env} = \frac{dp}{dz}
\]

Therefore, we have

\[
\frac{dw}{dt} = -\frac{1}{\rho_{par}} \left( \frac{dp}{dz} \right)_{env} - g = -\frac{1}{\rho_{par}} (-\rho_{env}g) - g = g \left( \frac{\rho_{env}}{\rho_{par}} - 1 \right)
\]

Therefore, a parcel starting at rest will accelerate upward (downward) if it is less (more) dense than the surrounding air, as expected. Using the ideal gas law, we can rewrite this expression as
Therefore, a parcel starting at rest will accelerate upward (downward) if it is warmer (cooler) than the surrounding air, as expected. Making use of a Taylor series expansion:

\[
\frac{dw}{dt} = g \left( \rho_{env} - \rho_{par} \right) = g \left[ \frac{p}{RT_{env}} - \frac{p}{RT_{par}} \right] = g \left[ \frac{1}{T_{env}} - \frac{1}{T_{par}} \right] = g \left[ \frac{T_{par} - T_{env}}{T_{env}} \right]
\]

where it is assumed that we are examining a dry air parcel

\[
T_{env} = T_0 + \left( \frac{dT}{dz} \right)_{env} \delta z + \cdots = T_0 - \Gamma \delta z + \cdots
\]

\[
T_{par} = T_0 + \left( \frac{dT}{dz} \right)_{par} \delta z + \cdots = T_0 - \Gamma_d \delta z + \cdots
\]

Similarly, for a saturated air parcel, we have

\[
\frac{dw}{dt} = g \left[ \frac{T_{par} - T_{env}}{T_{env}} \right] \approx g \left[ \frac{(T_0 - \Gamma_d \delta z) - (T_0 - \Gamma \delta z)}{T_{env}} \right] = g \left[ \frac{\Gamma - \Gamma_d}{T_{env}} \right] \delta z
\]

Therefore, we have two basic conditions

\[
\Gamma > \Gamma_d \Rightarrow \frac{dw}{dt} > 0
\]

\[
\Gamma > \Gamma_s \Rightarrow \frac{dw}{dt} < 0
\]

which are the conditions of absolute instability and absolute stability, as mentioned previously. Recall that for an unsaturated parcel,
A similar expression can be derived for a saturated parcel. The definition of equivalent potential temperature is given by

\[
\theta_e = \theta \exp \left[ \frac{L_v w_s}{c_p T} \right]
\]

Taking the natural logarithm and differentiating with respect to height gives

\[
\frac{1}{\theta_e} \frac{d\theta_e}{dz} = \frac{1}{\theta} \frac{d\theta}{dz} + \frac{L_v}{c_p} \frac{d}{dz} \left( \frac{w_s}{T} \right) = \frac{\Gamma_d - \Gamma}{T} + \frac{L_v w_s}{c_p T} \frac{d}{dz} \frac{w_s}{T} - \frac{L_v w_s}{c_p T^2} \frac{dT}{dz} = \frac{\Gamma_d - \Gamma}{T} - \frac{L_v w_s}{c_p T} \frac{d}{dz} T + \frac{L_v w_s}{c_p T^2} \frac{dT}{dz}
\]

Rearranging gives

\[
\frac{T}{\theta_e} \frac{d\theta_e}{dz} = \Gamma_d - \Gamma \left( 1 + \frac{L_v w_s}{c_p \frac{dT}{dz}} \right) + \frac{L_v w_s}{c_p T} \Gamma = \Gamma_d - \Gamma \frac{\Gamma_d}{\Gamma_s} + \frac{L_v w_s}{c_p T} \Gamma
\]

If the third term is considered small compared to the remaining two terms, we have

\[
\frac{T}{\theta_e} \frac{d\theta_e}{dz} \approx \Gamma_d \left( 1 - \frac{\Gamma}{\Gamma_s} \right)
\]

Therefore, when \( \Gamma_d < \Gamma \Rightarrow d\theta/dz < 0 \) and \( dw/dt > 0 \), which corresponds to an unstable environment. In this case, the parcel will accelerate away from its original position when displaced. When \( \Gamma_s > \Gamma \Rightarrow d\theta_e/dz > 0 \) and \( dw/dt < 0 \), which corresponds to a stable environment. In this case, the parcel will move back to its original position, regardless of whether or not saturation occurs. Lastly, the condition for conditionally instability is \( d\theta_e/dz < 0 \) and \( d\theta/dz > 0 \).
10.3: Thermodynamic Diagrams

Another way to assess the static stability of the atmosphere is to look at the vertical structure of the atmosphere at a specific location known as a vertical sounding. These vertical soundings are typically measured by an ascending radiosonde. A convenient way to visualize vertical soundings is to analyze thermodynamic diagrams. The most common thermodynamic diagram used in operational meteorology is the Skew-T, Log-P diagram. An example is given below.

A typical skew-T diagram contains five curves:

1. **Isobars** (i.e. lines of constant pressure): Isobars are lines that run horizontally from left to right and are labeled on the left side of the diagram. Pressure is given in increments of 100 mb and ranges from 1050 mb to 100 mb. Spacing between the isobars increases in the vertical because of the log scale that is used to represent pressure.
2. **Isotherms** (i.e. lines of constant temperature): Isotherms are the straight lines that slope from the bottom left to the upper right across the diagram. Increments are per degree and are labeled for every 5°C and the isotherms are labeled at the bottom of the diagram.

3. **Saturation mixing ratio lines:** Saturation mixing ratio lines are the slightly curved lines that slope from the lower left to the upper right that represent lines of equal mixing ratio. They are labeled on the bottom of the diagram in grams per kilogram of water vapor. They extend only to 200 mb and the spacing between them decreases as their values increase.

4. **Dry adiabats** (i.e. lines of constant \( \theta \)): Dry adiabats are the slightly curved, solid lines that slant from lower right to upper left. They are labeled every 10°C and indicate the dry adiabatic lapse rate (i.e. the rate of change of temperature in an air parcel of dry air rising or descending adiabatically).

5. **Moist adiabats or pseudoadiabats** (i.e. lines of constant \( \theta_e \)): Moist adiabats are lightly curved, solid lines sloping from lower right to upper left. They are labeled every 2°C and indicate the saturated adiabatic lapse rate (i.e. the rate of change of temperature in a saturated air parcel as it rises). They become parallel to the dry adiabats at the top of the charge because of the very low moisture content at those levels and stop at 200 mb.

Typically, the environmental profile of a Skew-T, Log-P diagram consists of a dew point curve and temperature curve, as shown in Figure 10.6. There are many parameters that are not directly reported on a sounding that can be calculated by using the lines discussed previously. We will discuss these in the following subsection

**10.3.1: Basic Sounding Parameters**

There are five basic parameters which can be determined from any sounding:

- Lifting condensation level (LCL)
- Convective condensation level (CCL)
- Convective temperature (\( T_c \))
- Level of free convection (LFC)
- Equilibrium level (EL)

Recall that the LCL is the level at which a parcel of air first becomes saturated when lifted dry adiabatically. The LCL can be found on a skew-T diagram from the following basic procedure (based on Figure 10.7)

1. From the dewpoint curve at the given pressure level, follow a line upward along a saturation mixing-ratio line.
2. From the temperature curve at the given pressure level, follow a line upward along a dry adiabat.
3. The intersection of these two lines is the LCL.

The convective temperature is the surface temperature that must be reached to start the formation of convective clouds by surface heating. The convective temperature can be found on a skew-T diagram from the following basic procedure (based on Figure 10.7)

1. Determine the CCL
2. From the CCL on the temperature curve, follow a dry adiabat downward to the surface
3. The temperature at this intersection is the convective temperature.

Figure 10.6 The Skew-T, Log-P diagram for Reno, Nevada (KUNR) on 2 February 2011 at 1200 UTC
Recall that the LFC is the height at which a lifted air parcel becomes warmer than the environmental air. The LFC can be found on a skew-T diagram from the following basic procedure (based on Figure 10.8)
1. Find the LCL
2. Follow the moist adiabat up to where it intersects the temperature curve.
3. This point is the LFC

Recall that the EL is the height where the temperature of a positively buoyant air parcel becomes equal to that of the environment, at which a lifted air parcel becomes warmer than the environmental air. The EL can be found on a skew-T diagram from the following basic procedure (based on Figure 10.8)
1. Find the LFC
2. From the LFC, follow the moist adiabat up to where it intersects the temperature curve.
3. This point is the EL
In the next section, we will analyze the basic convective indices that can be found on a skew-T, log-P diagram.

**10.3.2: Basic Convective Indices**

Another way to assess the static instability in the atmosphere is through the **convective available potential energy**, also known as **CAPE**. CAPE is the amount of energy an air parcel would have if lifted a certain distance vertically through the atmosphere. Essentially, CAPE is the positive buoyancy of an air parcel. It is proportional to the kinetic energy that a parcel can gain from its environment as a result of the contribution of the buoyancy to the vertical acceleration. On a Skew-T log-P diagram, it is often referred to as the **positive area** bounded by the environmental temperature profile and the parcel temperature profile, as shown in Figure 10.9. Therefore, we can define CAPE as

\[
CAPE = \int_{LFC}^{EL} B \, dz \approx g \int_{LFC}^{EL} \frac{T'_v}{T_v} \, dz
\]

where \(T_v\) is the virtual temperature of the environmental air and \(T'_v\) is the difference between the virtual temperature of the environmental air and the air parcel. CAPE can be found on a skew-T diagram from the following basic procedure (based on Figure 10.8)
1. Find the LFC
2. From the LFC, follow the air parcel along a moist adiabat until it intersects with the EL.
3. The area to the left of the temperature between the LFC and EL is the CAPE.

As mentioned above, in order for an air parcel to rise in the atmosphere, it must be lifted by some mechanical forcing process until it reaches its LFC. A measure of the amount of energy needed to accomplish this is called **convective inhibition**, also known as CIN. CIN is the amount of energy required to overcome the negative buoyant energy that the environment exerts on an air parcel. In other word, CIN is equal to the work that must be done against the stratification to lift a parcel of air to its LFC. On a Skew-T log-P diagram, it is often referred to as the **negative area** bounded by the environmental temperature profile and the path taken by a negatively buoyant air parcel from the surface to the LFC. Therefore, we can define CIN as

\[
CIN = \int_0^{LFC} \frac{B}{g} dz \approx g \int_{SFC}^{LFC} \frac{T' v}{T_v} \, dz
\]

CIN can be found on a skew-T diagram from the following basic procedure (based on Figure 10.8)

1. Find the LCL
2. From the LCL, follow the air parcel along a moist adiabat until it intersects with the LFC.
3. The area under the LFC and to the right of the temperature curve is CIN.

The other basic variables that can be used to assess convection are:

1. Relative humidity (RH)
2. Potential temperature (\(\theta\))
3. Equivalent potential temperature (\(\theta_e\))
4. K-index (K)
5. Lifted index (LI)
6. Showalter index (SI)
7. Total Totals index (TT)

RH can be found on a skew-T diagram from the following basic procedure

1. To determine the mixing ratio \(q\), read the value of the saturation mixing ratio that crosses the **dew point curve** at that pressure.
2. To determine the saturation mixing ratio \(q_s\), read the value of the saturation mixing ratio that crosses the **temperature curve** at that pressure.
3. Compute the relative humidity from \(RH = 100\% \times \frac{q}{q_s}\)
\( \theta \) can be found on a skew-T diagram from the following basic procedure from Figure 10.9

1. From the temperature curve at a given pressure level, follow the dry adiabat that interests
   the temperature curve down to 1000 mb
2. The isotherm value at this intersection is the potential temperature of the parcel at the
   given pressure.

\( \theta_e \) can be found on a skew-T diagram from the following basic procedure from Figure 10.10

1. Find the saturation level of the parcel from the pressure level of interest.
2. Follow a moist adiabat upward to a pressure level where the moist and dry adiabats are
   parallel.
3. Follow a dry adiabat down to the original pressure
4. From this point, follow a dry adiabat down to 1000 mb.
The K-index is used for determining what the probability and spatial coverage of ordinary thunderstorms would be based on temperature and dew point. The K-index is given by \( K = T_{850} + T_{d,850} + T_{700} - T_{500} \). The TT index gives an indication for the probability for seeing severe thunderstorm activity. The TT index is given by \( TT = T_{850} + T_{d,850} - 2T_{500} \). If storms do form, the lifted index (LI) is an index that indicates the severity of the storms. The LI is given by \( LI = T_{500} - T_{p,500} \). The relative values (and their importance) for these indices are given in Figure 10.11.
### K-INDEX

<table>
<thead>
<tr>
<th>K-value lower bound</th>
<th>K-value upper bound</th>
<th>Thunderstorm Coverage</th>
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</thead>
<tbody>
<tr>
<td>less than 20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>Rare</td>
</tr>
<tr>
<td>26</td>
<td>30</td>
<td>Isolated</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>Widely Scattered</td>
</tr>
<tr>
<td>greater than 35</td>
<td></td>
<td>Scattered</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Numerous</td>
</tr>
</tbody>
</table>

### TT INDEX

<table>
<thead>
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<th>TT-value lower bound</th>
<th>TT-value upper bound</th>
<th>Severe Thunderstorm Probability</th>
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</thead>
<tbody>
<tr>
<td>less than 44</td>
<td></td>
<td>Unlikely</td>
</tr>
<tr>
<td>44</td>
<td>48</td>
<td>Scattered, non-severe</td>
</tr>
<tr>
<td>48</td>
<td>52</td>
<td>Few severe</td>
</tr>
<tr>
<td>greater than 52</td>
<td></td>
<td>Many severe</td>
</tr>
</tbody>
</table>

### LIFTED INDEX (LI)

<table>
<thead>
<tr>
<th>LI Lower Bound</th>
<th>LI Upper Bound</th>
<th>Storm Severity</th>
</tr>
</thead>
<tbody>
<tr>
<td>greater than -2</td>
<td>-3</td>
<td>Weak</td>
</tr>
<tr>
<td>-5</td>
<td>Strong</td>
<td></td>
</tr>
<tr>
<td>greater than -5</td>
<td></td>
<td>very strong</td>
</tr>
</tbody>
</table>

Figure 10.10 The relative values for K index, TT index, and LI index
10.4: The Role of Synoptic-Scale Forcing on Convective Initiation

What we shall refer to as *deep moist convection* (DMC) arises when air is lifted to saturation and subsequently achieves positive buoyancy, such that it may rise to great heights. In other words, the initiation of DMC, which we shall simply refer to as *convection initiation*, requires that air parcels reach their level of free convection (LFC) and subsequently remain positively buoyant over a significant upward vertical excursion. Thus, *convective available potential energy (CAPE)* is a necessary, albeit insufficient, condition for convection initiation.

The location and timing of convection initiation is of acute interest to forecasters owing to the obvious association between convective storms and severe weather, in addition to less obvious impacts, such as the effects of convection on energy demand, future numerical weather predictions, and subsequent convective storm development. Dramatic warm season weather forecast failures are often the result of an inability to anticipate the initiation of DMC, or forecasting DMC that fails to develop. Convection initiation forecasting skill arguably has advanced at a slower rate than our ability to anticipate convective storm type, organization, and associated severe weather threats. Pinpointing where and when convective storms are likely to be initiated is a complex function of vertical motions ranging from the scale of thermals to the synoptic scale, mesoscale temperature and moisture inhomogeneities, and the mean stratification that largely results from synoptic-scale processes.

The presence of an LFC and CAPE requires a relatively large lower to middle tropospheric lapse rate (larger than the saturated adiabatic lapse rate, on average) and lower tropospheric moisture. The difficulty in accurately predicting convection initiation stems from the fact that the presence of CAPE is not a sufficient condition for convection initiation. Air typically requires some forced ascent in order to reach the LFC, owing to the presence of at least some convective inhibition (CIN) on most environmental soundings. Deep moist convection is commonly initiated along air mass boundaries such as synoptic fronts, drylines, outflow boundaries, and sea breezes. Convective storms can also be initiated by orographic circulations driven by the heating of elevated or sloped terrain, by forced lifting by ducted gravity waves, and by surface convergence near the center of a midlatitude cyclone.

Synoptic-scale dynamics often prime the mesoscale environment for convection initiation by way of large-scale mean ascent, which tends to reduce CIN and deepen the low-level moist layer. On the other hand, synoptic-scale dynamics also can discourage convection initiation by way of mean subsidence, which has the opposite effects. Large-scale vertical motions arguably can be anticipated reasonably well from pattern recognition and application of synoptic meteorology principles (e.g., quasigeostrophic theory), in conjunction with numerical model...
guidance. Synoptic-scale processes typically cannot be overlooked in making forecasts of convection initiation, despite the fact that convective initiation tends to be a largely mesoscale process.

Although the process of getting parcels to their LFC is an intrinsically mesoscale process, the synoptic scale processes sets the stage by modulating the CAPE and CIN, accomplished in part by modifications of the lapse rate. It can be shown that there are three synoptic-scale processes that can change that environmental lapse rate over time in a given region: (1) differential temperature advection, vertical lapse rate advection, divergence/convergence, and differential diabatic heating.

![Figure 10.11 Analysis of the environmental temperature difference between 500 and 700 mb, which is a bulk measure of the midlevel lapse rate](image)

The effects of horizontal lapse rate advection are seen in Figure 10.11. Here, large lapse rates (a temperature difference of 27 K between 500 and 700 mb corresponds to an approximately dry adiabatic environmental temperature profile) from the high terrain of northern Mexico and eastern New Mexico are being advedced toward the southern Great Plains of the United States. This common warm season phenomenon leads to the formation of an elevated mixed layer that provides static stability in the Great Plains region. On the synoptic scale, this is the most important process that alters the environmental lapse rate.
Figure 10.12 Schematic thermodynamic diagram illustrating the effect of vertical lapse rate advection. The light blue arrows indicate dry adiabatic parcel displacements.

Figure 10.12 illustrates the effect of vertical lapse rate advection. When upward motion is imposed at level $z_1$, larger lapse rates are advected from below $z_1$ upward to $z_1$, increase the lapse rate there. Note that this process occurs adiabatically, so that cooling has occurred at $z_1$ in addition to increasing the lapse rate there. This cooling associated with upward motion is typically more important for thunderstorm initiation than just the increasing lapse rate. For example, dry adiabatic large-scale ascent always leads to cooling when lapse rates are less than dry adiabatic, but lapse rate changes resulting from large-scale ascent may or may not always be significant.

Figure 10.13 Schematic thermodynamic diagram illustrating the effect of differential horizontal temperature advection (by the ageostrophic wind) on the lapse rate (temperature changes are indicated by the light blue arrows).

Figure 10.13 illustrates the effect of differential horizontal temperature advection. Here, cold advection increases with height at level $z_1$, which leads to an increase in the lapse rate at that level (similar to the role of thermal advection in QG theory). Because of the effect of differential
horizontal temperature advection in altering the environmental lapse rate, hodograph analysis on thermodynamic diagram are vital for understanding the timing of convective initiation.

Figure 10.14 Schematic thermodynamic diagram illustrating the stretching effect on lapse rate. The light blue arrows indicate dry adiabatic upward parcel displacements.

Figure 10.14 illustrates the effect of the stretching term. In this particular example, $\Gamma < \Gamma_d$. Thus, if the vertical velocity increases with height, this will lead to an increase in the lapse rate at level $z_1$ in time. Another way to state this idea is that the vertical advection of temperature (by the ageostrophic wind) is less than the temperature changes due to adiabatic expansion above the level of interest, causing a cooling process at upper-levels and an increase in the adiabatic lapse rate.

Figure 10.15 Schematic thermodynamic diagram illustrating the effects of differential diabatic heating on lapse rate.

Figure 10.15 illustrates the effects of differential diabatic heating on lapse rate. In this example, the maximum latent heating occurs at level $z_1$. Hence, the lapse rate increases above the level of maximum heating and decreases below the level of maximum heating. On the synoptic-scale, this is typically the least important process in affecting lapse rates.
**10.4.1: Derivation of Lapse Rate Tendency Equation**

We begin with the first law of thermodynamics written as

\[ dQ = c_p \, dT - \alpha \, dP \Rightarrow \dot{Q} = c_p \frac{dT}{dt} - \alpha \frac{dP}{dt} \]

Assuming hydrostatic conditions, it follows that

\[ \dot{Q} = c_p \left( \frac{\partial T}{\partial t} + \vec{V} \cdot \nabla T + w \frac{\partial T}{\partial z} \right) + gw \]

Differentiating with respect to \( z \) gives

\[ -\frac{\partial \dot{Q}}{\partial z} = c_p \left[ \frac{\partial}{\partial t} \left( -\frac{\partial T}{\partial z} \right) + \vec{V} \cdot \nabla \left( -\frac{\partial T}{\partial z} \right) + w \frac{\partial}{\partial z} \left( -\frac{\partial T}{\partial z} \right) - \frac{\partial \vec{V}}{\partial z} \cdot \nabla T - \frac{\partial w}{\partial z} \frac{\partial T}{\partial z} \right] - g \frac{\partial w}{\partial z} \]

Making use of our definitions of the environmental lapse rate \( \Gamma \) and dry adiabatic lapse rate \( \Gamma_d \), we have

\[ \frac{\partial \Gamma}{\partial t} = -\vec{V} \cdot \nabla \Gamma - w \frac{\partial \Gamma}{\partial z} + \vec{V} \cdot \nabla T + \frac{\partial w}{\partial z} (\Gamma_d - \Gamma) - \frac{1}{c_p} \frac{\partial \dot{Q}}{\partial z} \]

This is known as the lapse rate tendency equation. The first and second terms on the right hand side are the horizontal and vertical lapse rate advection terms (Figure 10.11 and 10.12), respectively. The third term, when combined with the horizontal lapse rate advection, represents the effects of differential temperature advection (Figure 10.13):

\[ -\vec{V} \cdot \nabla \Gamma - \frac{\partial \vec{V}}{\partial z} \cdot \nabla T = -\frac{\partial}{\partial z} (-\vec{V} \cdot \nabla T) \]

The fourth term is sometimes called the *stretcing* term. When \( \partial w/\partial z > 0 \) (as would be the case below the level of nondivergence when rising motion is present) and \( \Gamma < \Gamma_d \), the term acts to increase the environmental lapse rate (Figure 10.14). The term vanishes when \( \Gamma = \Gamma_d \) because the vertical advection of temperature cancels the temperature changes owing to adiabatic expansion/compression both above and below the level of interest, allowing the level to maintain a dry adiabatic lapse rate. The fifth term on the right hand side represents differential diabatic heating. If diabatic heating decreases (increases) with height, the lapse rate is increased (decreased), with the opposite being true for cooling.